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Abstract. In this paper we develop a unifying Markov-switching GARCH model which enables us (1) to specify complex GARCH equations in two distinct Markov-regimes, and (2) to model GARCH equations of different functional forms across the two Markov-regimes. To give a simple example, our flexible Markov-switching approach is capable of estimating an exponential GARCH (EGARCH) specification in the first and a standard GARCH specification in the second Markov-regime. We derive a maximum likelihood estimation framework and apply our general Markov-switching GARCH model to daily excess returns of the German stock market index DAX. Our empirical study has two major findings. First, our estimation results unambiguously indicate that our general model outperforms all conventional Markov-switching GARCH models hitherto estimated in the financial literature. Second, we find significant Markov-switching in the German stock market with substantially differing volatility structures across the regimes.

JEL classification: C5, G10, G15

Keywords: Markov-switching models; GARCH models; Dynamics of stock index returns

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1 Introduction

Since the seminal papers of Engle (1982) and Bollerslev (1986) GARCH (generalized autoregressive conditional heteroskedasticity) models have become a standard tool in modeling the conditional variances of the returns from financial time series data. The popularity of these models stems from (1) their compatibility with major stylized facts for asset returns, (2) the existence of efficient statistical methods for estimating model parameters, and (3) the availability of useful volatility forecasts.

In order to cover specific volatility features like the well-known leverage effect and other asymmetries in financial returns (e.g. Black, 1976; Christie, 1982; Schwert, 1989), a plethora of GARCH specifications have been suggested in the literature among the most prominent being the exponential GARCH (EGARCH) model introduced by Nelson (1991) and the threshold GARCH (TGARCH) model of Zakoian (1994). However, Hentschel (1995) establishes a connection between many of these models by showing that their specifications are special cases of a Box-Cox (1964) transformation to the conditional standard deviation.

While all the above-mentioned single-regime GARCH specifications have been well-established from a statistical point of view and have become standard routines in many econometric software packages, their two-regime Markov-switching counterparts are less straightforward to implement. Apart from the (typically) large number of parameters that have to be estimated this lack may be due to a phenomenon known as *path dependence* which stems from the GARCH lag structure and causes the regime-specific conditional variance to depend on the entire history of the data in a Markov-switching GARCH model. As pointed out by Cai (1994) and Hamilton and Susmel (1994) path dependence typically entails severe estimation problems if not carefully handled. However, Gray (1996) establishes a path-independent Markov-switching GARCH framework that permits direct estimation of all model parameters using (quasi) maximum likelihood techniques. Gray's model was later refined by Klaassen (2002) and it is their Markov-switching framework that we will expand in this paper.

Today, Markov-switching (or regime-switching) GARCH models, which are designed to capture discrete shifts in the volatility process of time series data, are in widespread use in various fields of financial economics. Most recent empirical applications of Markov-switching GARCH models to commoditiy prices, stock returns and exchange-rate return data are presented, *inter alia*, in Alizadeh et al. (2008), Henry (2009), Wilfling (2009) and Bohl et al. (2011). However, all two-regime Markov-switching GARCH specifications hitherto estimated in the economics and financial literature have one feature in common that appears unnecessarily restrictive. Despite the fact that the parameters in the variance equations are allowed to switch across both regimes, the overall functional forms of the two regime-specific GARCH equations are modeled as identical. For example, apart from Henry (2009) all authors of the above-cited empirical applications specify two-regime Markov-switching models with standard GARCH equations in each Markov regime while Henry (2009) uses EGARCH specifications in both regimes.

In this paper we develop a more flexible setup by incorporating Hentschel's (1995) results on the nesting of distinct symmetric and asymmetric single-regime GARCH models into Gray's (1996) Markov-switching GARCH model. In this way, we establish a general regime-switching framework that enables us to estimate complex GARCH equations of different functional forms across the Markov regimes. To give an example, our setup allows us to specify an EGARCH equation in regime 1 while regime 2 might be described by a standard GARCH specification. To our best knowledge such a flexible Markov-switching GARCH framework has not yet been implemented in the literature. In the empirical part of the paper we apply our general Markov-switching GARCH approach to the excess returns generated by the German stock index DAX and demonstrate that our flexible setup econometrically outperforms all conventional Markov-switching GARCH models hitherto estimated in the financial literature.

The remainder of the paper is organized as follows. Section 2 formally establishes our general Markov-switching GARCH framework. For ease of readability we derive the complete maximum likelihood estimation procedure in the technical appendix to the paper. Section 3 describes the data set and presents the estimation results. The final Section 4 summarizes the main results and concludes the paper.

2 A general Markov-switching GARCH model

In this section we establish our general Markov-switching model that enables us to specify and estimate GARCH equations of different functional forms in each of the distinct Markov regimes. For this we assume that the data generating process (DGP) of the financial return r_t is affected by an unobserved latent random variable S_t representing the regime the DGP is in at time t. For simplicity we assume only the two distinct regimes 1 and 2 at any point in time, that is, we assume either $S_t = 1$ or $S_t = 2$ for all $t = 1, 2, \ldots$.

As a starting point of our derivation we will follow Hentschel's (1995) exposition and build up the so-called *Absolute Value GARCH* model for the return process $\{r_t\}$. However, we expand Hentschel's single-regime framework to a two-regime Markov-switching model. To this end we let the return dynamics depend on the regime indicator $S_t = i, i = 1, 2$ and specify

$$r_{t+1} = \lambda_i + \gamma_i \sqrt{h_{i,t}} + \sqrt{h_{i,t}} \epsilon_{t+1}. \tag{1}$$

In Eq. (1), λ_i and γ_i are regime-specific constants while $\{\epsilon_t\}$ denotes an i.i.d. process of standard normal variates. $h_{i,t}$ represents the conditional variance in regime i the modeling of which will be treated below. The term $\lambda_i + \gamma_i \sqrt{h_{i,t}}$ on the right-hand side of Eq. (1) constitutes the mean equation of the return in regime i and is known as the GARCH-in-Mean (GARCH-M) model suggested by Engle et al. (1987) which has been used in many empirical studies on the behavior of stock returns (e.g. Elyasiani and Mansur, 1998; Ghysels et al., 2005). Based on these assumptions, the conditional distribution of the return is a mixture of two normal distributions which can be written as

$$r_{t+1}|\phi_t \sim \begin{cases} N(\lambda_1 + \gamma_1 \sqrt{h_{1,t}}, h_{1,t}) & \text{with probability} \quad p_{1,t} \\ N(\lambda_2 + \gamma_2 \sqrt{h_{2,t}}, h_{2,t}) & \text{with probability} \quad (1 - p_{1,t}) \end{cases}. \tag{2}$$

In Eq. (2) ϕ_t defines the information set as of date t and $p_{1,t} \equiv \Pr\{S_t = 1|\phi_t\}$ denotes the so-called *ex-ante* probability of being in regime 1 at date t. It is instructive to note that the information set ϕ_t basically coincides with the return path $\tilde{r}_t = \{r_t, r_{t-1}, \ldots\}$, but does not contain the path of the unobservable regime indicator S_t.

In the modeling of our regime-specific GARCH equations, we follow the path-independent methodology developed in Gray (1996).[1] In order to circumvent the aforementioned problem of path dependence, we specify the dynamics of the regime-specific conditional variance $h_{i,t}$ in terms of a lagged variance h_{t-1} and a shock term δ_t which are both appropriately weighted aggregates of the past conditional variances $h_{1,t-1}$ and $h_{2,t-1}$ from both Markov regimes. At this point we make use of an econometric improvement on Gray's approach suggested by Klaassen (2002). Klaassen's idea is to exploit all available information when integrating out the unobserved regimes in order to establish the aggregated variances and shock terms while Gray uses only part of it. To be more precise, in specifying the volatility $h_{i,t}$ valid in regime i, Klaassen computes the aggregated variance h_{t-1} and the shock terms δ_t on the basis of probabilities which explicitly take into account that we consider regime i at time t. This modeling improvement is particularly efficient when the Markov regimes appear to be highly persistent. In order to indicate the use of this additional information we denote the aggregated variance for date $t-1$ conditional on the fact that we are in regime i on date t by $h_{t-1}^{(i)}$, and accordingly the shock terms by $\delta_t^{(i)}$. In particular, we specify both quantities as

$$\begin{aligned} h_{t-1}^{(i)} &= p_{1,t-1}^{(i)} h_{1,t-1} + (1 - p_{1,t-1}^{(i)}) h_{2,t-1} \\ &\quad + p_{1,t-1}^{(i)} (1 - p_{1,t-1}^{(i)}) \left[\lambda_1 + \gamma_1 \sqrt{h_{1,t-1}} - (\lambda_2 + \gamma_2 \sqrt{h_{2,t-1}}) \right]^2 \end{aligned} \tag{3}$$

and

$$\delta_t^{(i)} = p_{1,t-1}^{(i)} \frac{r_t - (\lambda_1 + \gamma_1 \sqrt{h_{1,t-1}})}{\sqrt{h_{1,t-1}}} + (1 - p_{1,t-1}^{(i)}) \frac{r_t - (\lambda_2 + \gamma_2 \sqrt{h_{2,t-1}})}{\sqrt{h_{2,t-1}}}, \tag{4}$$

respectively, where the probabilities $p_{1,t-1}^{(i)}$ are calculated from Eq. (A.13) in the Appendix.[2]

Based on the aggregated variance and shock terms $h_{t-1}^{(i)}$ and $\delta_t^{(i)}$ from the Eqs. (3) and (4),

[1]It worth mentioning that technically speaking Gray's Markov-switching GARCH framework constitutes a collapsing procedure which facilitates the evaluation of the likelihood function at the cost of introducing a negligable approximation error. For an alternative approach to Markov-switching GARCH models see Haas et el. (2004).

[2]Instead of using the more informative Klaassen probabilities $p_{1,t-1}^{(i)}$, Gray (1996) uses the *ex-ante* probabilities $p_{1,t-1}$ from Eq. (2). This implies that Gray's aggregated variances and shock terms are equal irrespective of the Markov regime considered at date t.

we now define a first preliminary two-regime conditional volatility equation as

$$\sqrt{h_{i,t}} = \sqrt{Var(r_{t+1}|\phi_t, S_t = i)} = \omega_i + \alpha_i \sqrt{h_{t-1}^{(i)}} |\delta_t^{(i)}| + \beta_i \sqrt{h_{t-1}^{(i)}}, \tag{5}$$

where ω_i, α_i and β_i denote regime-specific volatility parameters to be estimated from the data. It is obvious that the volatility equation (5) constitutes a standard GARCH(1,1) model in which the conditional variance terms and the shock terms have been replaced by the conditional standard deviations and the absolute shock terms, respectively. However, an important drawback of this volatility equation is its incapability of capturing empirically well-documented asymmetries in the volatility of financial returns. In order to resolve this deficit, we follow Hentschel (1995), who generalizes the volatility equation (5) in a single-regime framework, and specify the second version of our two-regime conditional volatility equation as

$$\sqrt{h_{i,t}} = \omega_i + \alpha_i \sqrt{h_{t-1}^{(i)}} f_i(\delta_t^{(i)}) + \beta_i \sqrt{h_{t-1}^{(i)}} \tag{6}$$

with

$$f_i(\delta_t^{(i)}) = |\delta_t^{(i)} - b_i| - c_i(\delta_t^{(i)} - b_i), \tag{7}$$

where b_i, c_i represent regime-specific parameters. In what follows, we refer to the Eqs. (6) and (7) as the *Absolute Value GARCH* (AVGARCH) model.

Although the AVGARCH specification is interesting in its own right, Hentschel (1995) demonstrates that a Box-Cox (1964) transformation of the conditional standard deviation in the Eqs. (6) and (7) produces a rich class of models that includes many well-known symmetric and asymmetric GARCH models as special cases. Adapting this approach to our two-regime Markov-switching framework by introducing the regime-specific parameters μ_i and ν_i, we transform our conditional volatility Eq. (6) to

$$\frac{\sqrt{h_{i,t}}^{\mu_i} - 1}{\mu_i} = \omega_i + \alpha_i \sqrt{h_{t-1}^{(i)}}^{\mu_i} f_i^{\nu_i}(\delta_t^{(i)}) + \beta_i \frac{\sqrt{h_{t-1}^{(i)}}^{\mu_i} - 1}{\mu_i}. \tag{8}$$

The parameter μ_i determines the shape of the Box-Cox transformation in regime i. For $0 \leq \mu_i \leq$

1 the transformation of the conditional standard deviation $\sqrt{h_{i,t}}$ is concave while it is convex for $\mu_i > 1$. The parameter ν_i transforms the regime-specific function $f_i(\cdot)$. For $0 < \nu_i < 1$ the function $f^{\nu_i}(\cdot)$ becomes concave on either side of b_i while it becomes convex for $\nu_i > 1$. A convenient choice of the parameter c_i on the right-hand side of Eq. (7) is $|c_i| \leq 1$ since this condition guarantees a positive value of $f^{\nu_i}(\delta_t^{(i)})$. However, $|c_i| \leq 1$ is neither a necessary nor a sufficient condition to ensure $\sqrt{h_{i,t}} \geq 0$. Table 1, compiled from Table 1 in Hentschel (1995, p. 79), reveals how our volatility Eq. (8) for regime i nests many GARCH models scattered in the literature by imposing appropriate restrictions on the parameters μ_i, ν_i, b_i and c_i.[3]

Table 1 about here

Finally, we close our econometric model by specifying the probabilistic nature of the regime indicator S_t. In our study we let $\{S_t\}$ follow a two-state first-order Markov process with time-varying transition probabilities and write this as

$$\begin{aligned}
\Pr(S_t = 1|S_{t-1} = 1, r_t) &= P_t, \\
\Pr(S_t = 2|S_{t-1} = 1, r_t) &= 1 - P_t, \\
\Pr(S_t = 1|S_{t-1} = 2, r_t) &= 1 - Q_t, \\
\Pr(S_t = 2|S_{t-1} = 2, r_t) &= Q_t.
\end{aligned} \tag{9}$$

The probability of being in regime i for $i = 1, 2$ depends on realizations in $\tilde{r}_t$ and $\{S_t\}$ only through S_{t-1}. For the time-varying transition probabilities we assume

$$\begin{aligned}
P_t &= \Phi(d_1 + e_1 \cdot r_t), \\
Q_t &= \Phi(d_2 + e_2 \cdot r_t),
\end{aligned} \tag{10}$$

with $\Phi(\cdot)$ denoting the cumulative distribution function of a standard normal variate and d_1, d_2, e_1, e_2 representing parameters to be estimated from the data.

Our Markov-switching GARCH model established in the Eqs. (1) to (10) can now be estimated using (quasi) maximum likelihood techniques. The log-likelihood function is constructed recursively and we present its exact form in the Eqs. (A.1) to (A.14) of the Appendix. In the

[3]Recently, alternative single-regime GARCH models, which are not nested by our volatility Eq. (8), have been developed and applied to option-pricing problems. For an overview see Kim et al. (2010).

next section we apply this general Markov-switching GARCH framework to the daily excess returns of the German stock index DAX.

3 Empirical application

3.1 Data

We now analyze the mean and volatility structure of the daily excess returns sampled from the German stock market between 3 January 2000 and 31 December 2009 (2554 observations). We construct the excess returns r_t by subtracting an appropriately defined risk-free interest rate from the returns of the German stock index DAX.[4] Our DAX returns used for calculating the excess returns are adjusted for dividend payments. As the risk-free interest rate we use the *Euro OverNight Index Average* EONIA which we convert into daily returns by dividing the given annualized EONIA rate by 250.[5]

Figure 1 about here

Figure 1 displays the German stock index DAX (upper panel) and the corresponding DAX excess returns r_t (lower panel) during the sampling period. The trajectory of the excess returns clearly exhibits the two most prominent features well-documented in the financial literature on asset-return dynamics, namely volatility clustering and a time-varying mean. We now turn to analyzing these dynamic structures within our Markov-switching GARCH framework developed in Section 2.

Table 2 about here

[4] Our interest-rate data is provided by the *Deutsche Bundesbank* while we obtain the stock-market data from *Datastream* (daily closing prices).

[5] We divide by 250 in order to be consistent with the approximate number of observations per year available for the DAX returns.

3.2 Estimation results

Table 2 displays the maximum-likelihood (ML) estimates of five distinct Markov-switching GARCH models represented by the Eqs. (1) to (10). We numerically maximized the log-likelihood functions from the Eqs. (A.1) to (A.14) by the use of the BFGS-algorithm as implemented in the FMINCON module of the software package MATLAB. Our estimation results are robust to different starting values. To circumvent numerical problems stemming from the absolute value function appearing on the right-hand side of Eq. (7), we follow Hentschel (1995) and replace the argument of the absolute value function by a hyperbolic approximation.[6] Standard errors were computed from the diagonal of the heteroskedasticity-consistent (White-robust) covariance matrix.

Our Markov-switching GARCH framework developed in Section 2 is so general that it enables us to specify and estimate a large number of distinct two-regime Markov-switching GARCH models. Restrictions on the regime-dependent parameters μ_i, ν_i, b_i and c_i may lead to specific functional forms of the two variance equations, for example to an EGARCH equation in regime 1 ($\mu_1 = 0, \nu_1 = 1, b_1 = 0, c_1 =$ free) and a standard GARCH equation in regime 2 ($\mu_2 = 2, \nu_2 = 2, b_2 = 0, c_2 = 0$). In what follows, we refer to this latter model as a Markov-switching EGARCH-GARCH model and, based on the terminology in Table 1, we analogously use the phrasing TGARCH-GARCH, EGARCH-EGARCH and so on. Because of space constraints, we confine ourselves to estimating five distinct two-regime Markov-switching specifications for the DAX excess returns, namely (1) a standard GARCH-GARCH model, (2) an AVGARCH-AVGARCH model, (3) an EGARCH-GARCH model, (4) an EGARCH-EGARCH model, and (5) a so-called Free-Free model without any parameter restrictions.

The parameter estimates and standard errors for our five Markov-switching GARCH specifications reported in Table 2 can be used to assess the statistical significance of the model parameters. To this end, we consider the conventional t-statistic the exact finite-sample distribution of which is generally unknown in our estimation setup. However, we can make asymp-

[6]Technical details on the estimation procedure are available upon request.

totic inference by noting (1) that our ML estimators are asymptotically normally distributed, and (2) that our standard errors constitute (weakly) consistent estimates of the true standard deviations of the ML estimators. Consequently, under the null hypothesis of a single parameter being equal to 0, our t-statistics should converge in distribution towards a standard normal variate implying critical values of 1.6449, 1.9600 and 2.5758 at the 10, 5, and 1% levels, respectively, for the absolute value of the t-statistic (see Greene 2008, Appendix D). Following this reasoning, we find (1) that all parameters are statistically significant at least at the 10% level and (2) that the overwhelming majority (namely 80 out 85) parameters are significant at the 1% level.

An important econometric issue concerns the persistence of volatility shocks. In a standard single-regime GARCH(1,1)-equation of the form $h_t = \omega + \alpha \cdot h_{t-1}\delta_t^2 + \beta h_{t-1}$, the persistence of volatility shocks is typically measured by the sum $\alpha + \beta$. The higher the value of $\alpha + \beta$, the longer it takes until a volatility shock dies out. In particular, when $\alpha + \beta = 1$ volatility shocks have a permanent effect and the unconditional variance of the process gets infinitely large. In view of these considerations within a single-regime framework, it appears natural to measure the persistence of volatility shocks in a two-regime Markov-switching GARCH(1,1) model by the regime-specific sums $\alpha_i + \beta_i$ for $i = 1, 2$. Unfortunately, matters turn out to be more complicated, since in general it is the interaction between the regime-specific volatility parameters and the transition probabilities of the regime indicator S_t which determines the variance-stability of a Markov-switching GARCH model.[7]

Since exact mathematical conditions covering the variance-stability of Markov-switching GARCH models are not available in the literature, we are restricted to analyzing the persistence of volatility shocks within each Markov regime. From Column 1 of Table 2 we find that the respective regime-specific sums $\hat{\alpha}_i + \hat{\beta}_i$ for our Markov-switching GARCH-GARCH model are given by 0.9857 and 0.9873 indicating covariance stationarity with high degrees of volatility persistence in both Markov-regimes. A very similar result holds for regime 2 of our Markov-

[7]See for example, Wilfling (2009) and the literature cited there

switching EGARCH-GARCH model (Column 3 of Table 2) for which we find $\hat{\alpha}_2 + \hat{\beta}_2 = 0.9884$. For the most general Markov-switching Free-Free model a sufficient condition for covariance stationarity in regime i is given by

$$E\left[[\alpha_i \cdot \mu . f^{\nu_i}(\epsilon_t) + \beta_i]^{2/\mu_i}\right] < 1 \tag{11}$$

(see Nelson, 1990). Hentschel (1995) shows that for an AVGARCH specification with $\mu_i = \nu_i = 1$ condition (11) is equivalent to

$$\begin{aligned} &\alpha_i^2(1+b_i^2)(1+c_i^2) + \beta_i^2 + 2\alpha_i\beta_i b_i c_i + 4\alpha_i(\beta_i + \alpha_i b_i c_i)\phi(b_i) \\ &+ 2\alpha_i(\beta_i b_i + \alpha_i(1+b_i^2)c_i)(2\Phi(b_i) - 1) < 1, \end{aligned} \tag{12}$$

with $\phi(\cdot)$ and $\Phi(\cdot)$ denoting the probability density and cumulative distribution functions of the standard normal distribution, while for a regime-specific EGARCH equation condition (11) converges to

$$\beta_i < 1. \tag{13}$$

For both AVGARCH regimes in our second Markov-switching specification the estimates from Column 2 of Table 2 yield the values 0.9778 an 0.9684 when inserted into the left-hand side of condition (12) thus again indicating covariance stationarity with high degrees of volatility persistence in both Markov-regimes. An analogous empirical result obtains for all EGARCH Markov-regimes for which we find estimates of the parameters β_1 and β_2 that are all close to but smaller than 1. Only for the Markov-switching Free-Free specification there is no closed-form solution to the expectation on the left-hand side of condition (11). However, we calculated this expectation by numerical integration again finding evidence of covariance stationarity and high volatility persistence in both Markov-regimes.

Our time-varying transition probabilities P_t and Q_t from Eq. (10) represent the likelihood that no switch in the Markov-regimes occurs between the dates $t-1$ and t. In all of our 5 Markov-switching specifications the probabilities P_t and Q_t are larger than 0.97 at (nearly) every point in time indicating an extremely high degree of regime persistence.

Next, we address several specification issues. As a first diagnostic check we may test for first- and higher-order serial correlation of the squared standardized residuals. To this end we performed Ljung-Box-Q-tests for serial correlation out to various lags for our five Markov-switching specifications. The tests do not reveal any statistical evidence in favor of autocorrelation in the residuals except for the GARCH-GARCH specification for which higher-order serial correlation is detected.[8]

An important specification issue concerns the number of Markov-regimes modeled in our regime-switching representation (1) – (10). Testing the significance of a second Markov-regime is a non-trivial task due to an identification problem known as the Davies Problem (see Davies, 1987). The identification problem implies that a conventional likelihood ratio test (LRT) may be statistically improper since we cannot assume the validity of the χ^2-approximation to the LRT statistic under the null hypothesis of a single Markov-regime any longer. However, Gelman and Wilfling (2009) assess the finite-sample properties of the conventional LRT statistic (defined as twice the difference in the log-likelihoods of the two-regime Markov-switching and the single-regime specifications) for a GARCH-GARCH model by a parametric bootstrapping procedure. Their results indicate that the null distribution of the LRT statistic typically does not exhibit large deviations from the χ^2-distribution with degrees of freedom equal to the difference in the number of parameters between the two-regime and the single-regime specifications. Encouraged by these simulation results, we have conducted the conventional LR tests for all our five Markov-switching specifications. In all cases the LRT statistics are so extreme that they exceed all critical values used in practice thus endorsing our two-regime specifications estimated in Table 2.[9]

Next, we address the question as to which of our five alternative Markov-switching specifications provides the best fit to the data. Obviously, we cannot test all models against each other since two distinct specifications need to be nested in order to assure a likelihood ratio test to be valid. Since our Markov-switching Free-Free model nests all the other specifications (see Ta-

[8]Details of the autocorrelation tests are available upon request.

[9]Details of the LR tests are available upon request.

ble 1), we restrict attention to the four testing problems (1) 'H_0: GARCH-GARCH versus H_1: Free-Free', (2) 'H_0: AVGARCH-AVGARCH versus H_1: Free-Free', (3) 'H_0: EGARCH-GARCH versus H_1: Free-Free' and (4) 'H_0: EGARCH-EGARCH versus H_1: Free-Free'.

Table 3 about here

Table 3 displays the log-likelihood values of all Markov-switching specifications along with the LRT statistics of the four testing problems just mentioned. Obviously, the LR tests clearly reject the GARCH-GARCH, the AVGARCH-AVGARCH and the EGARCH-EGARCH models against the Free-Free model at significance levels far below the 1% level. Only for the EGARCH-GARCH model the specification testing results are slightly less clear-cut. The p-value 0.0315 indicates that the EGARCH-GARCH model is rejected against the Free-Free model at the 5%, but not at the 1% level.

However, some technical remarks on this latter testing problem are in order. To this end, consider for a moment a single-regime EGARCH and a single-regime Free model. Although theoretically the EGARCH model is nested within the Free model class, testing the EGARCH model against the Free model may cause practical problems. The reason is that in order to guarantee a positive standard deviation for the Free model, we implemented a lower bound for the parameters ω, α and β at zero. Within the Free model class these parameter restrictions ensure positive standard deviations for all models with $\mu \neq 0$. Theoretically, for specifications within the Free model class with $\mu = 0$ these restrictions are no longer necessary. However, when estimating the Free model specification we retained the parameter restrictions for ω, α and β to (1) facilitate numerical optimization, and (2) to be capable of computing standard errors of our estimates. By contrast, when estimating an EGARCH specification with $\mu = 0, \nu = 1, b = 0$, we followed standard practice and did not impose the (unnecessary) restrictions on the parameters ω, α and β. Since in this setting the Free model does not really nest the EGARCH model, it is theoretically possible that a two-regime Markov-switching model with an EGARCH specification in at least one regime might have a higher log-likelihood value than the alternative Free-Free model. From a probabilistic point of view this implies an increased Type II error of the test and thus a lower power of the test.

Figure 2 about here

Figure 2 displays the *ex-ante* regime-1 probabilities calculated according to Eq. (A.7) along with the conditional variances of the daily excess returns of the German stock market index DAX as estimated by our five Markov-switching GARCH specifications. For all five models the conditional variances exhibit a strikingly uniform pattern during the sampling interval between the years 2000 and 2010. The beginning of the decade started with a period of relatively high volatility in the German stock market with a pronounced peak in conditional variances around 11 September 2001. After a short phase of normalization, an extended period of high stock-market volatility occurred between mid-2002 and the end of 2003 reflecting the German bear market in which the DAX fell from about 5000 to 2000 index points. Between 2004 and the beginning of the year 2008 the conditional volatility of the DAX was comparably low. This period of low market fluctuation came to an abrupt end at the beginning of the year 2008 when the German stock market began to respond to the subprime crisis by plummeting stock prices. However, the highest volatility peak occurred around 15 September 2008 when Lehman Brothers Holdings Inc. filed for *Chapter 11 bankruptcy protection*.

Analyzing the *ex-ante* regime-1 probabilities in Figure 2, we find that all our five Markov-switching models generate two or more pronounced regime switches. Some of these regime switches appear to occur at the same time irrespective of the chosen Markov-switching specification. The most clear-cut example is the switch at the end of the year 2008 possibly indicating a structural change in the German excess returns since the financial crisis. Four out of five specifications—including our EGARCH-GARCH and Free-Free models—report a regime switch around June 2006 when a sustained bullish trend in the German stock market began. Obviously, the regimes 1 and 2 estimated via the *ex-ante* probabilities of our five Markov-switching models do not necessarily coincide with the low- and high-volatility periods depicted in the neighboring panels. A first explanation of this finding is that each Markov-switching specification allows for both, a switching mean and a switching volatility equation, so that a regime-switch may solely be induced by a switch in the mean equation alone. A second explanation is that each regime-specific variance specification is capable of capturing certain qualitative volatility features (e.g. specific volatility asymmetries) which do not directly affect the volatility level, but which may nevertheless induce a structural switch from one regime to the other.

However, the most efficient way of investigating switching volatility structures is to analyze the Free-Free model, which clearly outperforms all other specifications. Within this model class we can test for pairwise equality of the corresponding regime-specific volatility parameters (i.e. $\mu_1 = \mu_2, \nu_1 = \nu_2, \omega_1 = \omega_2, \alpha_1 = \alpha_2, \beta_1 = \beta_2, b_1 = b_2, c_1 = c_2$). Apart from the parameters β_1 and β_2, all other corresponding volatility parameter appear to be considerably different from each other across both regimes thus indicating substantial structural differences between both volatility regimes in the German stock index DAX.

4 Summary and conclusion

In this paper we establish a two-regime Markov-switching GARCH model which enables us to estimate complex functional GARCH specifications within each regime. Combining Gray's (1996) and Klaassen's (2002) Markov-switching framework with Hentschel's (1995) approach of nesting alternative single-regime GARCH models, our framework unifies many Markov-switching GARCH models that have been estimated hitherto in the financial literature. Apart from complex regime-specific GARCH specifications, our model features two further empirically relevant attributes, namely (1) a GARCH-in-Mean specification of the mean equation, and (2) time-varying transition probabilities describing the dynamics of the latent regime-indicator.

In the technical appendix to the paper, we develop a reliable maximum likelihood estimation algorithm for our model which we apply to appropriately constructed daily excess returns of the German stock index DAX for the time between January 2000 and December 2009. Our empirical analysis reveals that our model unambiguously outperforms alternative Markov-switching GARCH models applied so far in the literature. Moreover, we find significant Markov-switching in the German stock market with substantially differing volatility structures across both Markov-regimes.

A natural line of future research could be the extension of our framework to more than two Markov-regimes. This, however, leads to highly parameterized models which become increasingly difficult to estimate. However, other estimation procedures than our ML approach may be implemented, for example Bayesian Markov Chain Monte Carlo (MCMC) algorithms which have the potential to provide an alternative way of circumventing the problem of path dependence (see Bauwens et al., 2010).

Appendix A. Maximum likelihood estimation

In this appendix we construct the log-likelihood function for our Markov-switching GARCH model established in Section 2. We only consider the two-regime case although a theoretical extension of the entire framework to more Markov regimes is straightforward.

The conditional probability distribution of r_{t+1} is shown in Eq. (2). The corresponding probability density function has the form

$$\begin{aligned} f(r_{t+1}|\phi_t) &= \sum_{i=1}^{2} f(r_{t+1}, S_t = i|\phi_t) \\ &= \sum_{i=1}^{2} \Pr(S_t = i|\phi_t) \cdot f(r_{t+1}|S_t = i, \phi_t) \\ &= \sum_{i=1}^{2} p_{i,t} \cdot f(r_{t+1}|S_t = i, \phi_t), \end{aligned} \tag{A.1}$$

where, as in the main text, $p_{i,t} \equiv \Pr(S_t = i|\phi_t)$ denotes the *ex-ante* regime-i probability. The information set ϕ_t consists of the entire history of $\tilde{r}_t = \{r_t, r_{t-1}, \ldots\}$.

Since the regime indicator S_t follows a first-order Markov process the *ex-ante* probability

$p_{i,t}$ depends only on S_{t-1} and r_t. Using the *Theorem of Total Probabilities*, we obtain

$$p_{i,t} = \sum_{j=1}^{2} \Pr(S_t = i | S_{t-1} = j, \tilde{r}_t) \Pr(S_{t-1} = j | \tilde{r}_t). \tag{A.2}$$

The first probability $\Pr(S_t = i | S_{t-1} = j, \tilde{r}_t)$ on the right-hand side of (A.2) does not depend on the entire history of $\tilde{r}_t$ so that we replace $\tilde{r}_t$ by r_t in this latter probability. Thus, we can insert the probabilities specified in Eq. (9) in Eq. (A.2) and obtain

$$\begin{aligned} p_{1,t} &= P_t \cdot \Pr(S_{t-1} = 1 | \tilde{r}_t) + (1 - Q_t) \cdot \Pr(S_{t-1} = 2 | \tilde{r}_t) \\ &= P_t \cdot \Pr(S_{t-1} = 1 | \tilde{r}_t) + (1 - Q_t) \cdot (1 - \Pr(S_{t-1} = 1 | \tilde{r}_t)), \end{aligned} \tag{A.3}$$

and analogously

$$p_{2,t} = Q_t \cdot (1 - \Pr(S_{t-1} = 1 | \tilde{r}_t)) + (1 - P_t) \cdot \Pr(S_{t-1} = 1 | \tilde{r}_t). \tag{A.4}$$

The remaining probability $\Pr(S_{t-1} = 1 | \tilde{r}_t)$ in the Eqs. (A.3) and (A.4) can be written as a function of $p_{1,t-1} = \Pr(S_{t-1} = 1 | \tilde{r}_{t-1})$. To this end, we apply *Bayes' Formula* yielding

$$\begin{aligned} \Pr(S_{t-1} = 1 | \tilde{r}_t) &= \Pr(S_{t-1} = 1 | r_t, \tilde{r}_{t-1}) \\ &= \frac{f(r_t | S_{t-1} = 1, \tilde{r}_{t-1}) \Pr(S_{t-1} = 1, \tilde{r}_{t-1})}{\sum_{i=1}^{2} f(r_t | S_{t-1} = i, \tilde{r}_{t-1}) \Pr(S_{t-1} = i, \tilde{r}_{t-1})}. \end{aligned} \tag{A.5}$$

Expanding the ratio on the right-hand side of Eq. (A.5), we obtain

$$\begin{aligned} \Pr(S_{t-1} = 1 | \tilde{r}_t) &= \frac{f(r_t | S_{t-1} = 1, \tilde{r}_{t-1}) \Pr(S_{t-1} = 1 | \tilde{r}_{t-1})}{\sum_{i=1}^{2} f(r_t | S_{t-1} = i, \tilde{r}_{t-1}) \Pr(S_{t-1} = i | \tilde{r}_{t-1})} \\ &= \frac{f(r_t | S_{t-1} = 1, \tilde{r}_{t-1}) p_{1,t-1}}{\sum_{i=1}^{2} f(r_t | S_{t-1} = i, \tilde{r}_{t-1}) p_{i,t-1}} \\ &= \frac{g_{1,t-1} \cdot p_{1,t-1}}{\sum_{i=1}^{2} g_{i,t-1} \cdot p_{i,t-1}}, \end{aligned} \tag{A.6}$$

where, for ease of notation, we have defined $g_{i,t-1} \equiv f(r_t | S_{t-1} = i, \tilde{r}_{t-1}) = f(r_t | S_{t-1} = i, \phi_{t-1})$. Using Eq. (A.6), we are now able to calculate the *ex-ante* probability $p_{1,t}$ by inserting Eq. (A.6) in Eq. (A.3):

$$\begin{aligned} p_{1,t} &= P_t \cdot \frac{g_{1,t-1}p_{1,t-1}}{g_{1,t-1}p_{1,t-1} + g_{2,t-1}(1-p_{1,t-1})} + (1-Q_t) \cdot \left[1 - \frac{g_{1,t-1}p_{1,t-1}}{g_{1,t-1}p_{1,t-1} + g_{2,t-1}(1-p_{1,t-1})}\right] \\ &= P_t \cdot \frac{g_{1,t-1}p_{1,t-1}}{g_{1,t-1}p_{1,t-1} + g_{2,t-1}(1-p_{1,t-1})} + (1-Q_t) \cdot \frac{g_{2,t-1}(1-p_{1,t-1})}{g_{1,t-1}p_{1,t-1} + g_{2,t-1}(1-p_{1,t-1})}. \end{aligned} \tag{A.7}$$

Next, we address the exact form of the conditional density f appearing in the Eqs. (A.1) and (A.7). As we are assuming conditional normality f is given as follows:

$$f(r_{t+1}|S_t = i, \phi_t) = \frac{1}{\sqrt{2\pi h_{i,t}}} \exp\left\{-\frac{\left[r_{t+1} - (\lambda_i + \gamma_i\sqrt{h_{i,t}})\right]^2}{2h_{i,t}}\right\}. \tag{A.8}$$

The variance $h_{i,t}$ depends on the explicit functional form of the GARCH equation. It is easy to check from Eq. (8) that it can be written as

$$h_{i,t} = \begin{cases} \left[\omega_i + \alpha_i \sqrt{h_{t-1}^{(i)}}^{\mu_i} f_i^{\nu_i}(\delta_t^{(i)}) + \beta_i \sqrt{h_{t-1}^{(i)}}^{\mu_i}\right]^{2/\mu_i} & \text{for} \quad \mu_i > 0 \\ \left[\exp\left\{\omega_i + \alpha_i f_i^{\nu_i}(\delta_t^{(i)}) + \beta_i \ln\left(\sqrt{h_{t-1}^{(i)}}\right)\right\}\right]^2 & \text{for} \quad \mu_i = 0 \end{cases}, \tag{A.9}$$

with appropiately defined parameters $\omega_i, \alpha_i, \beta_i$.

It is obvious from Eq. (A.9) that for the calculation of regime-specific variances $h_{i,t}$ we need the aggregated variances and shock terms $h_{t-1}^{(i)}$ and $\delta_t^{(i)}$ the calculation of which we base on the Klaassen (2002) probabilities $p_{1,t-1}^{(i)}$ as described in the main text. Using *Bayes' Formula* again, we obtain the Klaassen probabilities as

$$\begin{aligned} p_{1,t-1}^{(i)} &= \Pr(S_{t-1} = 1|\tilde{r}_{t-1}, S_t = i) \\ &= \frac{\Pr(S_t = i|\tilde{r}_{t-1}, S_{t-1} = 1)\Pr(S_{t-1} = 1|\tilde{r}_{t-1})}{\Pr(S_t = i|\tilde{r}_{t-1})} \\ &= \frac{\Pr(S_t = i|\tilde{r}_{t-1}, S_{t-1} = 1) \cdot p_{1,t-1}}{\Pr(S_t = i|\tilde{r}_{t-1})}, \end{aligned} \tag{A.10}$$

with $p_{1,t-1}$ as given in Eq. (A.7). Applying the *Theorem of Total Probabilities* once more, we write the denominator in Eq. (A.10) as

$$\begin{aligned}\Pr(S_t = i|\tilde{r}_{t-1}) &= \Pr(S_t = i|\tilde{r}_{t-1}, S_{t-1} = 1) \cdot p_{1,t-1} \\ &\quad + \Pr(S_t = i|\tilde{r}_{t-1}, S_{t-1} = 2) \cdot (1 - p_{1,t-1}). \qquad \text{(A.11)}\end{aligned}$$

To calculate the probability on the left-hand of Eq. (A.11) we need the two probabilities $\Pr(S_t = i|\tilde{r}_{t-1}, S_{t-1} = 1)$ and $\Pr(S_t = i|\tilde{r}_{t-1}, S_{t-1} = 2)$. To be consistent with the specifications (9) and (10) for the time-varying transition probabilities, we have to choose appropriate forecasts of the return r_t conditional on either $\tilde{r}_{t-1}, S_{t-1} = 1$ or $\tilde{r}_{t-1}, S_{t-1} = 2$. In what follows we use the conditional expectations $E(r_t|\tilde{r}_{t-1}, S_{t-1} = 1) = \lambda_1 + \gamma_1\sqrt{h_{1,t-1}}$ and $E(r_t|\tilde{r}_{t-1}, S_{t-1} = 2) = \lambda_2 + \gamma_2\sqrt{h_{2,t-1}}$ which are known to be optimal forecasts with respect to the mean squared error (MSE). Thus, we obtain

$$\begin{aligned}\Pr(S_t = 1|\tilde{r}_{t-1}, S_{t-1} = 1) &= \Phi\left(d_1 + e_1 \cdot \left[\lambda_1 + \gamma_1\sqrt{h_{1,t-1}}\right]\right), \\ \Pr(S_t = 2|\tilde{r}_{t-1}, S_{t-1} = 1) &= 1 - \Phi\left(d_1 + e_1 \cdot \left[\lambda_1 + \gamma_1\sqrt{h_{1,t-1}}\right]\right), \\ \Pr(S_t = 1|\tilde{r}_{t-1}, S_{t-1} = 2) &= 1 - \Phi\left(d_2 + e_2 \cdot \left[\lambda_2 + \gamma_2\sqrt{h_{2,t-1}}\right]\right), \\ \Pr(S_t = 2|\tilde{r}_{t-1}, S_{t-1} = 2) &= \Phi\left(d_2 + e_2 \cdot \left[\lambda_2 + \gamma_2\sqrt{h_{2,t-1}}\right]\right).\end{aligned} \qquad \text{(A.12)}$$

Now, inserting the Eqs. (A.12) and (A.11) in Eq. (A.10) we obtain

$$\begin{aligned}p^{(1)}_{1,t-1} &= \frac{\Phi(d_1 + e_1[\lambda_1 + \gamma_1\sqrt{h_{1,t-1}}])p_{1,t-1}}{\Phi(d_1 + e_1[\lambda_1 + \gamma_1\sqrt{h_{1,t-1}}])p_{1,t-1} + \{1 - \Phi(d_2 + e_2[\lambda_2 + \gamma_2\sqrt{h_{2,t-1}}])\}p_{2,t-1}}, \\ p^{(1)}_{2,t-1} &= 1 - p^{(1)}_{1,t-1}, \\ p^{(2)}_{1,t-1} &= \frac{\{1 - \Phi(d_1 + e_1[\lambda_1 + \gamma_1\sqrt{h_{1,t-1}}])\}p_{1,t-1}}{\{1 - \Phi(d_1 + e_1[\lambda_1 + \gamma_1\sqrt{h_{1,t-1}}])\}p_{1,t-1} + \Phi(d_2 + e_2[\lambda_2 + \gamma_2\sqrt{h_{2,t-1}}])p_{2,t-1}}, \\ p^{(2)}_{2,t-1} &= 1 - p^{(2)}_{1,t-1}.\end{aligned} \qquad \text{(A.13)}$$

Finally, we use the recursive structures developed so far to construct the log-likelihood function of our flexible Markov-switching model defined in the Eqs. (1) to (10). The general form of the likelihood function is

$$L(\mathbf{\Theta}) = f(r_t, \ldots, r_1; \mathbf{\Theta}),$$

with the vector $\mathbf{\Theta}$ containing all model parameters. Writing this joint distribution of the returns as a product of conditional densities, we obtain

$$L(\mathbf{\Theta}) = \prod_{t=1}^{T} f(r_t | \tilde{r}_{t-1}; \mathbf{\Theta}),$$

for which we define the starting term as $f(r_1|\tilde{r}_0; \mathbf{\Theta}) \equiv f(r_1; \mathbf{\Theta})$. Taking the logarithm of $L(\mathbf{\Theta})$ and inserting (the lagged form of) Eq. (A.1), we obtain the log-likelihood function as

$$\begin{aligned} \ell(\mathbf{\Theta}) \equiv \log[L(\mathbf{\Theta})] &= \sum_{t=1}^{T} \log\left[f(r_t|\tilde{r}_{t-1}; \mathbf{\Theta})\right] \\ &= \sum_{t=1}^{T} \log\left[\sum_{j=1}^{2} f(r_t|S_{t-1} = j, \tilde{r}_{t-1}; \mathbf{\Theta}) \cdot p_{j,t-1}\right]. \end{aligned} \tag{A.14}$$

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Figures and Tables

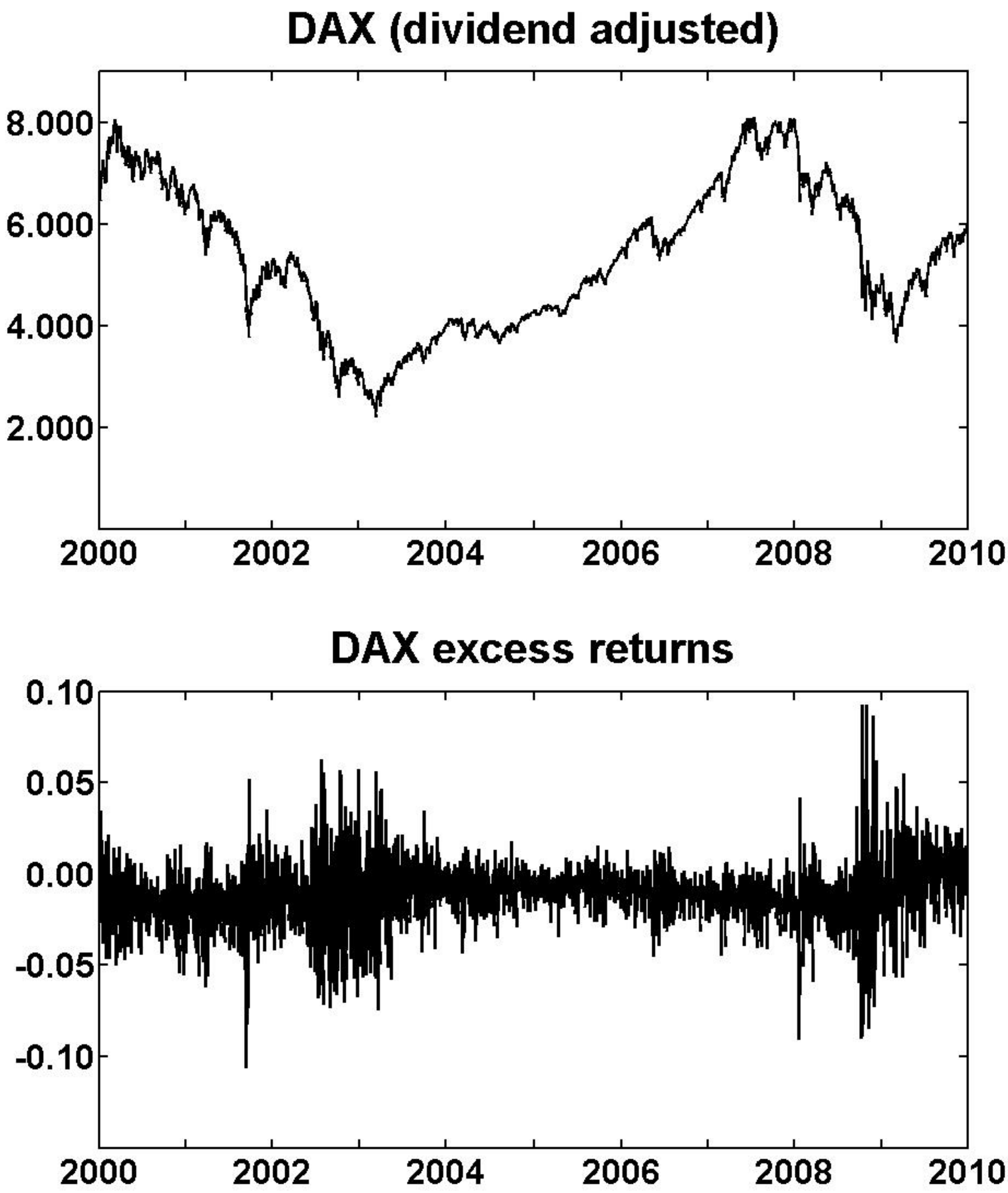

Fig. 1. Dividend adjusted DAX and DAX excess returns (2000 – 2009)

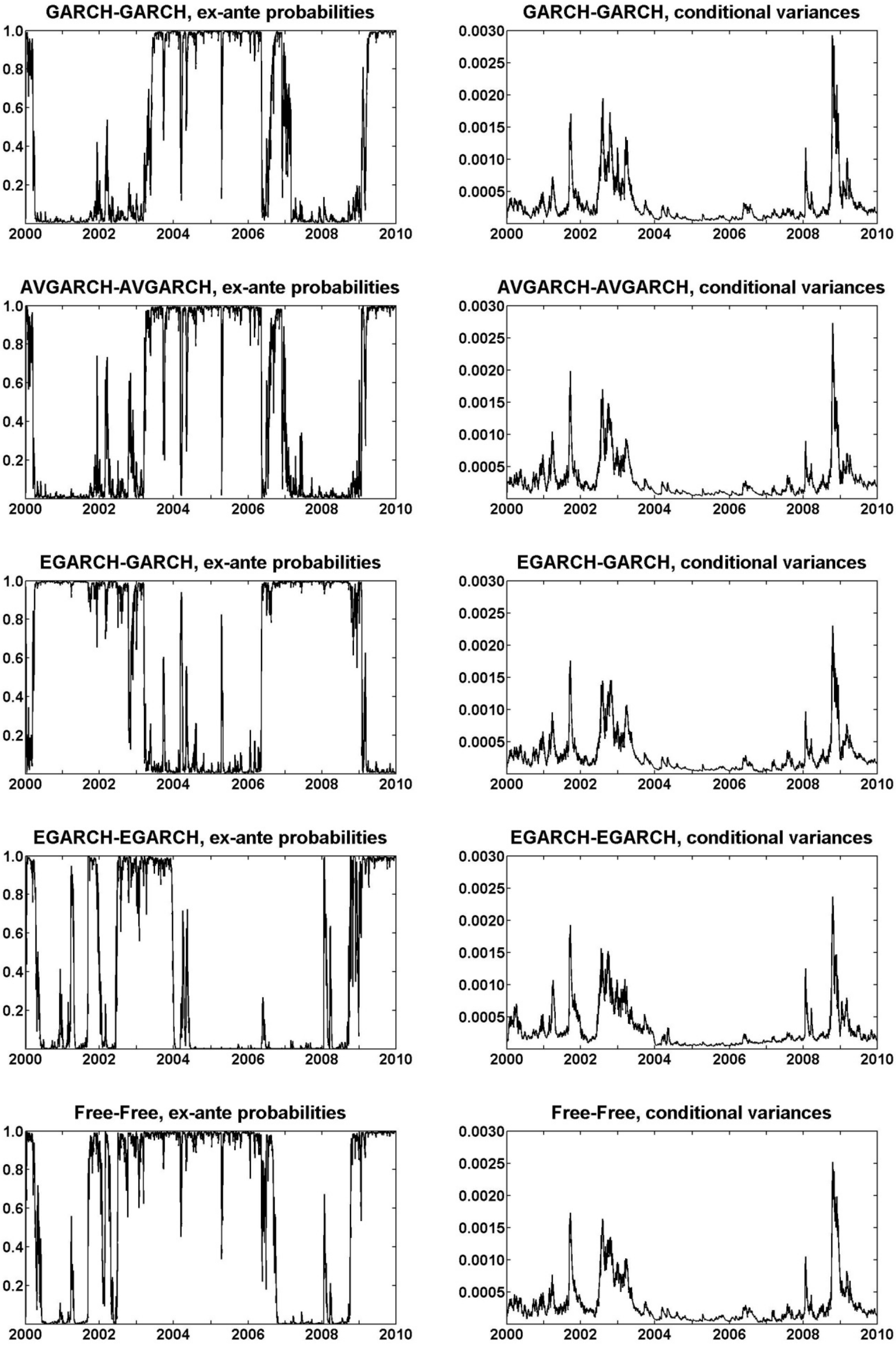

Fig. 2. *Ex-ante* regime-1 probabilities and conditional variances of five Markov-switching GARCH models

Table 1
Nested GARCH models (within regime i)

μ_i	ν_i	b_i	c_i	Model	Reference
0	1	0	free	Exponential GARCH (EGARCH)	Nelson (1991)
1	1	0	$\lvert c_i \rvert \leq 1$	Threshold GARCH (TGARCH)	Zakoian (1994)
1	1	free	$\lvert c_i \rvert \leq 1$	Absolute Value GARCH (AVGARCH)	Hentschel (1995)
2	2	0	0	Standard GARCH (GARCH)	Bollerslev (1986)
2	2	free	0	Nonlinear-asymmetric GARCH	Engle and Ng (1993)
2	2	0	free	Glosten-Jagannathan-Runkle GARCH	Glosten et al. (1993)
free	μ_i	0	0	Nonlinear ARCH	Higgins and Bera (1992)
free	μ_i	0	$\lvert c_i \rvert \leq 1$	Asymmetric power ARCH	Ding et al. (1993)

Note: Table compiled from Hentschel (1995, Table 1).

Table 2
Estimates of alternative Markov-switching GARCH specifications

	GARCH–GARCH	AVGARCH–AVGARCH	EGARCH–GARCH	EGARCH–EGARCH	Free–Free
μ_1	2.0000	1.0000	0.0000	0.0000	0.7597***
					(0.0156)
μ_2	2.0000	1.0000	2.0000	0.0000	0.0024**
					(0.0011)
ν_1	2.0000	1.0000	1.0000	1.0000	2.0126***
					(0.0053)
ν_2	2.0000	1.0000	2.0000	1.0000	1.4421***
					(0.0364)
λ_1	−0.0113***	−0.0120***	−0.0119***	0.0043***	−0.0071***
	(0.0006)	(0.0001)	(0.0001)	(0.0000)	(0.0007)
λ_2	−0.0176***	−0.0156***	−0.0108***	0.0028***	−0.0091***
	(0.0001)	(0.0001)	(0.0000)	(0.0000)	(0.0003)
γ_1	0.3853***	0.4403***	−0.2871***	−0.5740***	−0.0564***
	(0.0009)	(0.0011)	(0.0001)	(0.0000)	(0.0003)
γ_2	0.1285***	−0.0739***	0.3998***	−1.3937***	−0.6234***
	(0.0015)	(0.0001)	(0.0013)	(0.0000)	(0.0004)
ω_1	0.0000***	0.0001*	−0.1252***	−0.0989***	0.0003**
	(0.0000)	(0.0001)	(0.0002)	(0.0000)	(0.0002)
ω_2	0.0000*	0.0004***	0.0000***	−0.0600***	0.0250***
	(0.0000)	(0.0001)	(0.0000)	(0.0000)	(0.0003)
α_1	0.0594***	0.1081***	0.0741***	0.0365***	0.0259***
	(0.0008)	(0.0003)	(0.0001)	(0.0000)	(0.0005)
α_2	0.1219***	0.0896***	0.0464***	0.0108***	0.0001**
	(0.0024)	(0.0004)	(0.0001)	(0.0000)	(0.0000)
β_1	0.9263***	0.9647***	0.9839***	0.9819***	0.9624***
	(0.0043)	(0.0061)	(0.0005)	(0.0005)	(0.0056)
β_2	0.8654***	0.9082***	0.9420***	0.9886***	0.9743***
	(0.0208)	(0.0036)	(0.0029)	(0.0002)	(0.0006)
d_1	2.8417***	2.7771***	3.3509***	3.0705***	2.7773***
	(0.0473)	(0.0100)	(0.0036)	(0.0018)	(0.0687)
d_2	2.9047***	2.5439***	2.7844***	5.0867***	4.2702***
	(0.0368)	(0.0205)	(0.0052)	(0.0048)	(0.0645)
e_1	2.1697***	9.1372***	18.5418***	−36.9247***	−5.5716***
	(0.0215)	(0.0544)	(0.1223)	(0.0000)	(0.0011)
e_2	6.9612***	−11.8090***	−5.7178***	76.5404***	46.1943***
	(0.0440)	(0.0001)	(0.0001)	(0.0937)	(0.7260)
b_1	0.0000	1.2469***	0.0000	0.0000	0.6021***
		(0.0063)			(0.0071)
b_2	0.0000	0.0098***	0.0000	0.0000	2.5977***
		(0.0001)			0.0742)
c_1	0.0000	−0.9086***	0.7998***	1.4149***	−0.1538***
		(0.0001)	(0.0019)	(0.0011)	(0.0001)
c_2	0.0000	0.7351***	0.0000	3.8618***	−0.0319***
		(0.0018)		(0.0038)	(0.0001)

Note: Estimates for parameters from the Eqs. (1) to (10). Standard errors are in parentheses. *,** and *** denote statistical significance at 10, 5 and 1% levels, respectively.

Table 3

Log-likelihood values and likelihood ratio tests

	GARCH–GARCH	AVGARCH–AVGARCH	EGARCH–GARCH	EGARCH–EGARCH	Free–Free
Log-likelihood	7290.4170	7311.4635	7323.5311	7308.1564	7331.2173
LRT statistic vs Free-Free model	81.6006*** [0.0000]	39.5076*** [0.0000]	15.3724** [0.0315]	46.1218*** [0.0000]	
χ^2-df	8.0000	4.0000	7.0000	6.0000	

Note: The LRT statistic of the testing problem 'H_0: the considered two-regime specification versus H_1: the two-regime Free-Free specification' is computed as twice the difference in the log-likelihoods of the Free-Free specification and the two-regime specification under the null hypothesis. The LRT statistics are asymptotically χ^2-distributed under the respective null hypotheses with degree-of-freedom parameters as given in the row 'χ^2-df'. p-values are in squared brackets. *,** and *** denote statistical significance at 10, 5 and 1% levels, respectively.