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Optimal contract under asymmetric information: the role of options on futures

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Abstract

The aim of this paper is to show that an option on an appropriate future may solve some market failures caused by asymmetric information. Some models related to the adverse selection, moral hazard and verification costs are analyzed and the performance of these options on futures is evaluated. The typical situation regards a consumer (or an investor) who wishes to discount his/her future income in order to finance his/her present consumption (investment); under asymmetric information this agent may incur in liquidity constraints (credit rationing), which is not the case when buying the option on a futures contract. This contract is constructed so that the (future) agent's income is correlated with some futures contract (but this is private information) on which the option is issued. Some examples show that this is not a very stringent assumption.

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Key words: Asymmetric information, credit rationing, options on futures.

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Introduction

In economics, information asymmetry deals with the study of decisions in transactions where one agent has more or better information than the other. This creates an imbalance of power in transactions which can sometimes cause some market failure. Examples of this problem are adverse selection, moral hazard and costly monitoring. At present, information asymmetry is only a partially solved problem in economic theory although a vast literature has shown micro and macroeconomic effects of this phenomenon. In fact, asymmetric information may compromise the good functioning of the market in fixing the price of goods or services and causes serious consequences at the macroeconomic level. For example, if consumers or firms expect a high future income, they find it optimal to sell the stream of their future income to the banking system in order to obtain present cash flow (for consumption or investment, respectively); however, under asymmetric information, the market may fail: people who are willing to pay a price higher than the market price are rationed and consequently consumption or investment suffer from liquidity constraint. Put in another way, the traditional neoclassical economics literature assumes that markets are efficient except for some limited and well defined market failures. However, some authors argue that it is only under exceptional circumstances that markets are efficient. In 2001, the Nobel Prize in Economics was awarded to George Akerlof, Michael Spence, and Joseph E. Stiglitz "for their analyses of markets with asymmetric information".

This work proposes a micro-founded solution to the problem in question. The proposed solution relies on the market of options issued on futures (see Hull, chapter 16[2]). In this paper, as a benchmark, one considers a general model embedding either adverse selection or moral hazard or verification costs (which all imply credit rationing). One then shows that the option on futures provides an alternative way to access credit and (under certain conditions) this alternative does not suffer from market failure or credit rationing. The conditions whereby the option on a futures contract (O-F) is cheaper than a traditional debt contract (T-D) will be also shown. This is an important feature to show when borrowers are not "rationed". One then shows that borrowers with "good" expectations on their income always prefer an O-F contract instead of a T-D contract.

[2] In these contracts, the exercise of the option gives the holder a position in a futures contract.

A classic paper on adverse selection is George Akerlof's "The Market for Lemons" of 1970. George Akerlof notes that the average value of the commodity tends to go down, even for products of perfectly good quality. Because of information asymmetry, some sellers can hide the true quality of their items and defraud the buyer. As a result, buyers prove to be unwilling to purchase this commodity since this entails a certain amount of risk. Consequently, it is even possible for the market to decay to the point of nonexistence. Jaffe and Russell (1976) show that in the credit market, under asymmetric information between borrowers and lender, adverse selection may cause credit rationing. In a pooling equilibrium, lenders offer the same loan contract to all borrowers (who are either honest or dishonest) since they are unable to distinguish between the two types. The interest rate is calculated under the assumption that borrowers are drawn randomly from the population. But at this rate, the pool of borrowers is no longer the same as in the population: adverse selection occurs. In fact, the lender is more likely to attract the dishonest borrower and hence the lender´s expected profit tends to fall and the market may fail. However, loans are not characterized only by interest rates but also by the amount of some collateral. Given the collateral, equilibrium may imply credit rationing. In other words, credit rationing is the market response to adverse selection. Townsend (1979) provided the first analysis of optimal contracts when it is costly to verify the state. He shows that under the presence of asymmetric information between lender and borrower, a debt contract with costly state verification can be constructed so as to be Pareto efficient. Gale and Hellwig (1985) follow this strand and set the implication of a firm's investment level. Stiglitz and Weiss (1981) shed further light on the credit rationing due to the presence of adverse selection and moral hazard, showing that the profit function is hump-shaped with respect to the interest rate. For a survey of definitions of "credit rationing" see Walsh (2003, chapter 7). However, as Walsh argues: "The critical aspect of this definition is that, at the market equilibrium interest rate, there is an unsatisfied demand for loans that cannot be eliminated through higher interest rates". Williamson (1986, 1987a, 1987b) has illustrated how debt contracts and credit rationing can arise even in the absence of adverse selection or moral hazard, if lenders must incur costs to monitor borrowers. Gertler (1988) and Bernanke and Gertler (1989) show how agency costs drive a wedge between the costs of internal and external sources of financing; they further show how investment decisions also depend on the internal funds. Another huge strand of literature shows the relationship with credit rationing and the credit channel through which monetary factors and policy may affect the real economy. See Walsh

2003, chapter 8 for a survey. Moschini and Lapan (1995) outline a hedging role of Options on futures but they aimed to show that a combination of options on futures and futures contracts hedge a producer who faces production and price uncertainty. Instead, in this paper, one shows that (a portfolio of) options on futures may prevent the credit rationing and liquidity constraint.

The rest of this paper is organized as follows. The second section formally sets up notations and the assumptions of the economy; the third section shows how this O-F contract may work from the borrower's side. The fourth section shows, in a context of some models with market failures, the remedy provided by the O-F contract. The fifth section shows how a lender (bank) sets the optimal price of the T-D contract with verification costs and the price of the O-F contract. The sixth section shows the condition whereby a representative agent with non linear objective function, given the price of both contracts, chooses either contract. Conclusions then follow.

Notations and the assumptions in the economy

The following assumptions mostly recall the assumptions of Stiglitz and Weiss (1981) and of Williamson (1986, 1987a, 1987b).

a) The economy is composed of the futures, T-D, O-F, and risk-free bonds markets. The economy lasts 1, 2,..,N periods, N ≥3.
b) The futures market is complete and efficient: participants are rational, risk neutral, competitive and detain no private information. In this market, a continuum of futures are issued whose underlying value is denoted by X: X ∈ Ξ. The futures fix the price of each X_N in period 2; hence its price is the expected discount value of X_N.
c) In the T-D and in the O-F markets, there are identical lenders (banks) who are rational, risk neutral, have constant return to scale and operate competitively.
d) On the demand side of these markets, there is a continuum of rational borrowers (consumers or investors) indexed by θ: $\theta \in \Theta$, who evaluate an income stream Y in period N. θ may measure the a-priori degree of honesty of the borrowers (see Jaffe and Russell, 1976) or different attitudes of their ex-post behavior in affecting the project yielding Y_N.
e) Y is log-normally distributed such that: $\mu = E[\ln(Y)]$ and $\sigma^2 = Var[\ln(Y)]$. $F_j(Y_i)$ and $f_j(Y_i)$ are the distribution function and probability density function (respectively) of Y in period i conditional on information j, with j≤i.

f) At period 2, borrowers detain some private (but not perfect) information on $\mathrm{Y_N}$, banks do not.

g) Each income Y is correlated with one (combination of) X but this correlation is also private information of each borrower. Without loss of generality, it is assumed that this correlation is perfect. This implies that the borrower cannot exploit his private information about the correlation between Y and some X to obtain some funding (and the bank cannot infer the type (riskiness) of borrower through the knowledge of X). Assume also that each action of the borrower on Y is such that Y remains correlated with some other X, X ∈ Ξ.

h) If the borrower is a consumer, he wishes to discount the potential value $\mathrm{Y_N}$ in order to smooth his consumption (he is risk averter) in period 2; if he is an investor, he wishes to discount $\mathrm{Y_N}$ in order to finance his investment project yielding $\mathrm{Y_N}$. He also detains $\mathrm{Y_1}$ and $\mathrm{Y_2}$ in period 1 and 2 respectively.

i) The parameter θ (the type of borrowers) and the action of each borrower on the project yielding Y are not known by the lenders (or by other agents of the other markets) but they know the distribution function $H(\theta)$. This function describes the frequency of participation in the market of each type of borrower. However, this is affected by the price of each market since those borrowers who find it unprofitable exit the market, thereby altering the shape of $H(\theta)$. Assume that this process occurs within period 2.

j) In the T-D market, debt contracts are set in period 2 and redeemed in period N; the (competitive) market interest rate is $\mathrm{R_N^B}$ and each bank bids its optimal quantity of debt D (in period 2). Assume that this rate is neither a random variable nor D.

k) As regards the T-D contract, some (not mutually exclusive) alternatives are possible:
   - in period N, borrowers know $\mathrm{Y_N}$ for certain and without a cost, hence lenders have to pay a verification cost C;
   - lenders require a collateral c in order to grant the funds.

l) In the risk-free bonds market gross interest rates are denoted as $\mathrm{R_N} = (1 + \mathrm{r_{2,N}})^{\mathrm{N}-2}$ as the period 2 is the reference period in this economy. Assume that the term structure of interest rates is not a random variable.

m) In the O-F market, (call) option contracts are issued in period 1 and expire in period 2. Denote the price as CO_1, the quantity as Q and the multiple of the option as m. Each option gives the right to buy a futures contract at the strike price K. Since the value of each future is the discounted expected value of some X, and if Y_N is (perfectly) correlated with X_N , then the payoff of the option is, in period 2, $\frac{1}{m}\max(E_2[X_N]/R_N - K, 0) \equiv \frac{1}{m}\max(E_2[Y_N]/R_N - K, 0)$

The Option on Futures contract from the borrower's side

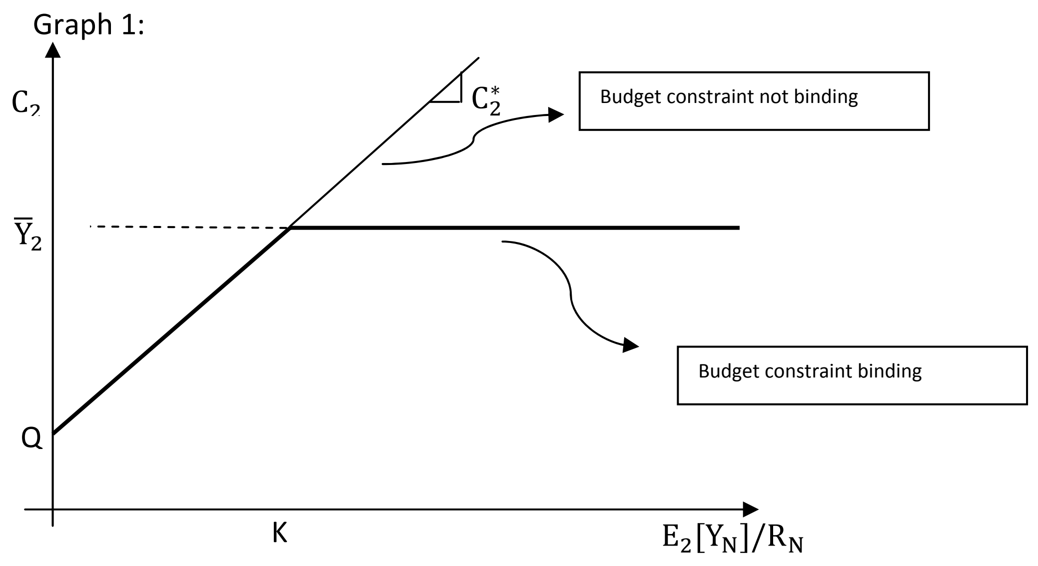
Consider an optimizing non risk-neutral consumer who seeks to smooth consumption over an N-times horizon period. Consider two cases in time 2. That is, he or she may or may not face liquidity constraints and, in the latter case, the constraint is binding. Suppose that income in time N is sufficiently high such that the consumer desires to discount it (to smooth consumption). Call this (desired) level of consumption $C_2^*(Y_N)$. This implies that if the consumer were constrained, consumption in time 2 is: $C_2^{\otimes} = Y_2 + R_2(Y_1 - C_1) < C_2^*(Y_N)$. Hence, in time 2, it holds:

$$C_2 = \min\left[C_2^*(Y_N), C_2^{\otimes}\right] \tag{1}$$

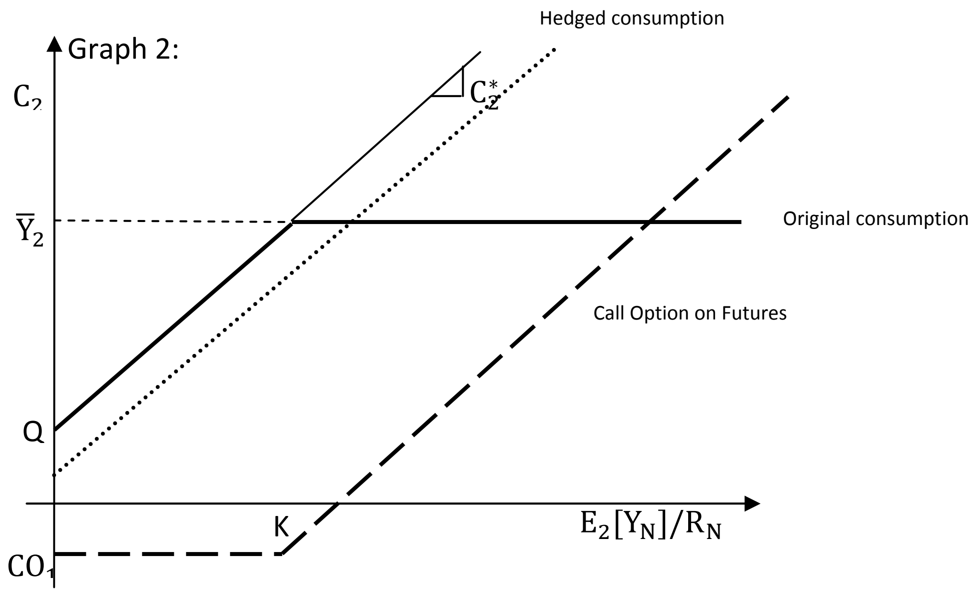
If the consumer has quadratic utility function, it can be shown that this outcome for C_2 has the same pay-off of a put-option written on Y_N. In fact, suppose now that $C_2^*(Y_N) = C_2^* Y_N$ $(C_2^* > 0)$ and that $C_2^* = 1$, then eq. (1) can be represented in graph 1.

This means that the consumption function in time 2 replicates a put option with strike price K. Intuitively, in time 2, the following cases may occur:

- the consumer faces no budget constraint: the consumption function is the line with slope C_2^* and intercept Q;
- the consumer faces a budget constraint but it is not binding: $E_2[Y_N]/R_N \leq K$; where K is the smallest value of Y_N such that the consumer desires to discount his or her future income;
- the consumer faces a binding budget constraint $E_2[Y_N]/R_N \geq K$ but it must be that: $C_2 = C_2^{\otimes}$



The structure outlined above shows the possibility of recovering a smoothed consumption in time 2 by buying (in time 1) a call option whose price is CO_1, with strike price K, written on a security correlated with Y_N, $E_2[Y_N]/R_N$; this security can be a futures contract whose "underlying" is (correlated with) Y_N. The effect of buying this kind of option is to obtain a (parallel) line with slope C_2^* and intercept Q-CO_1. See graph 2.



Simple mathematics shows that Q is a function of the interest rates and Y_1 and Y_2. The same example can be used to describe the situation of an entrepreneur with a profitable investment project yielding Y_N at time N where the internal funds of his firm are not enough to finance his project. The entrepreneur wants to discount the potential cash flow (derived from Y_N) in order to implement the desired investment. If he cannot access to some traditional debt (the firm is credit rationed), he can buy the O-F contract at period 1 in order to have his funding in time 2. Even if the firm is not rationed, the entrepreneur wants to minimize the cost of funding and hence compares the cost of the T-D contract with the O-F contract.

As a practical example, consider an economic agent who is confident about investing in a bakery that promises (in his/her opinion) a future high profit. The profit of this investment depends on the price of the bread (the output) and the price of some raw stuffs such as flour as input. Even if it is unrealistic to state that there is a futures market for the bakery, it is more realistic to assume the existence of a futures market for bread, flour and other raw stuffs. The economic agent should only seek a portfolio of options issued on futures of these stuffs. In particular, he/she should detain a call option on futures issued on bread and a put option on futures issued on flour.

Now suppose that a bank faces an agent (consumer or firm) who has private information about his/its future income. Consider, for example, traditional lending contracts that are affected by adverse selection. Following Jaffe and Russel (1976), suppose that agents are of two types: honest and dishonest. The first always states his/its information about future income whereas the second is truthful only if it is profitable to him/it. Supposing the bank knows for each type of borrower the probability that he is able to repay the debt, then the bank is able to set an interest rate that incorporates this information[3]. So, the presence of asymmetric information firstly implies inefficiency of the market because agents whose probability not to repay the debt is low, borrow at a higher interest rate; secondly and consequently, the latter agent may find it not profitable to borrow any more, hence adverse selection occurs.

Now consider the fact that the bank has the possibility to offer an O-F contract instead of a T-D contract. The question is now whether the agent may exploit his/its private information. Suppose the dishonest agent considers buying the O-F contract; anyone who buys an O-F contract must be long and hence cannot exploit his private information that his income may be low. Hence the O-F contract yields a “favorable selection” instead of an adverse selection. Note that, in all of the

[3] Suppose now that the bank cannot distinguish between them, hence the lending interest rate is an average of a low interest rate and a higher interest rate.

examples above, one has implicitly assumed that agents were long about their future discounted income.

The comparison of Options on Futures with the "Adverse Selection" , "Moral Hazard" and "Verification costs" models.

This section builds a comprehensive model including the Stiglitz and Weiss model (1981) of "adverse selection" and "moral hazard" and Williamson's model (1986, 1987a, 1987b) of "verification costs"[4]. One shows how the market failure can occur in this general model but this is not the case where the lender offers an O-F contract.

With the assumption that borrowers are a continuum labeled by θ, the expected profit of a bank when bidding a T-D contract is:

$$E_2[\Pi_N^B] = \int_{\theta:\,\theta\in\Theta} \left\{E_2^\theta\left[\min\left(Y_N + c, DR_N^B\right)\right] - CE_2^\theta\left[\frac{\min\left(Y_N + c - DR_N^B, 0\right)}{Y_N + c - DR_N^B}\right]\right\} dH(\theta) - R_N D \qquad (2)$$

If c=0 (c=collateral) then one has the Williamson model; if C=0 (C=verification costs) then one obtains the Stiglitz and Weiss models.

Note that, given the information structure between borrowers and lender, Townsend (1979) shows that the optimal contract that the bank is willing to offer to the borrower is a debt contract $E_2^\theta\left[\min\left(Y_N, DR_N^B\right)\right]$. He also shows that verification costs C are paid only in the case of bankruptcy (that is $Y_N + c < DR_N^B$). Note that this event may be described by the operator: $\frac{\min(Y_N + c - DR_N^B, 0)}{Y_N + c - DR_N^B}$.

For simplicity, assume (for now) that borrowers are a risk-neutral investor, then, the expected profit of each type of borrower θ is[5]

$$E_2[\Pi_N^\theta] = E_2^\theta[Y_N] - E_2^\theta\left[\min\left(Y_N, DR_N^B\right)\right] - cE_2^\theta\left[\frac{\min\left(Y_N - DR_N^B, 0\right)}{Y_N - DR_N^B}\right] \qquad (3)$$

[4] In these papers, Williamson has shown that credit rationing can arise even in the absence of adverse-selection or moral-hazard problems if lenders must incur costs to monitor borrowers.

[5] Note that eq. (3) is equivalent to the expected value of eq. (4a) of the Stiglitz and Weiss paper; however, eq. (3) of this paper highlights separately revenues and costs of the profit function.

Note that even if the collateral c is zero, the borrower may incur a loss because of the opportunity costs (not explicitly specified in eq. (3)) of investing his own capital in the project. Expectations in the more general model do depend on the action of each type of borrowers θ . Assume now that, at the beginning[6] of period 2, $\mathrm{E}_2^{\theta}[\mathrm{Y_N}] = \overline{\mathrm{E}_2^{\theta}[\mathrm{Y_N}]}\ \forall\ \theta\colon \theta \in \Theta$ but the $\mathrm{Var}_2^{\theta}[\mathrm{Y_N}]$ is not necessarily the same.

Consider now the case of the adverse selection (agents are different a priori but cannot change their behavior ex post, opportunistically). Assume that there are some borrowers who, given $\mathrm{f_2(Y_N)}$, c and some (initial) value of $\mathrm{R_N^B}$, find it not worthwhile to invest in the project ($\mathrm{E_2}[\Pi_\mathrm{N}^\mathrm{B}] < 0$). Since expected values are the same, this means that the variance of the project, which is different among borrowers, is the source of the profit negativity of some borrowers. One shows in appendix A that a higher variance[7] of the $\mathrm{Y_N}$ implies a lower value of the debt, namely: $\mathrm{E}_2^{\theta}\left[\min\left(\mathrm{Y_N}, \mathrm{DR_N^B}\right)\right]$. This means that only borrowers with a high variance are willing to accept the debt contract. It follows that the profits of the bank are now[8]:

$$\mathrm{E_2}[\Pi_\mathrm{N}^\mathrm{B}] = \int\limits_{\theta:\ \mathrm{E_2}[\Pi_\mathrm{N}^\mathrm{B}]>0} \left\{\mathrm{E}_2^{\theta}\left[\min\left(\mathrm{Y_N}, \mathrm{DR_N^B}\right)\right] - \mathrm{cE}_2^{\theta}\left[\frac{\min\left(\mathrm{Y_N} - \mathrm{DR_N^B}, 0\right)}{\mathrm{Y_N} - \mathrm{DR_N^B}}\right]\right\} \mathrm{dH}(\theta) - \mathrm{R_N D} \qquad (4)$$

Because of the different integration interval, these profits are less than the profits considered in Eq. (2). In order to maintain the desirable level of profit, banks are induced to raise $\mathrm{R_N^B}$ which increases[9] $\mathrm{E_2}\left[\min\left(\mathrm{Y_N}, \mathrm{DR_N^B}\right)\right]$ and causes a new round of adverse selection. All these adjustments occur in period 2; in equilibrium (at the end of period 2), market failure is possible as, for example, credit rationing.

Moral hazard works in a similar way: after getting the debt, some borrowers find it profitable to change their (original) project in order to have (positive) profits[10], hence changing projects with a

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[6] One assumes that at the beginning of period 2, $H(\theta)$ is not yet affected by the price of the relevant market.

[7] Note that since Y is log-normally distributed, the mean-preserving property (as in Stiglitz and Weiss (1981))does not apply.

[8] At the end of period 2 when $H(\theta)$ is affected by the initial price (interest rate) of the relevant market.

[9] See Appendix B.

[10] Since, at the beginning of period 2, they were not necessarily positive.

higher variance causes expected losses for the bank which, if it is able to foresee that, should exit this market; this again causes the market failure.

Note that if borrowers are characterized (or can affect) the revenue component of their profit functions instead of their cost component, as in the above situation, a "favorable" selection would be possible and the mechanism behind adverse selection and moral hazard would not work. This is the situation where borrowers evaluate buying an O-F contract. In this case, the profit of the borrower is

$$\mathrm{E_2}[\Pi_{\mathrm{N}}^{\theta}] = \mathrm{E_2}\left\{\mathrm{Y_N}\left[\frac{\mathrm{Q}}{\mathrm{m}}\max(\mathrm{E_2}[\mathrm{Y_N}]/\mathrm{R_N} - \mathrm{K}, 0)\right]\right\} - \mathrm{CO_1R_2Q} \tag{5}$$

where $\mathrm{Y_N}[.]$ is the income at period N as a function of the invested capital. Note now that $\mathrm{E_2}[\mathrm{Y_N}]$ does depend on the outcome of the O-F in period 2; as anticipated, the favorable selection applies here since the borrower must evaluate the possibility that the option ends up out-of-money or that its revenue when invested in the project is not enough to cover the costs $(\mathrm{CO_1R_2})$. Hence borrowers who have a bad evaluation of $\mathrm{E_2}[\mathrm{Y_N}]$ do not affect the profit of the bank apart from the fact that they do not buy the O-F contract.

The comparison with the costs of a T-D and an O-F contract

This section compares the price of this T-D contract (with costly state verification) with the price of the O-F contract. The aim is to show that the O-F is cheaper.

The optimal banking behavior when bidding a T-D contract

One has already argued that when the bank does not receive the promised payment $\mathrm{DR_N^B}$ it wants to incur in verification costs to see whether the investor is cheating. It can be shown that, under this kind of contract, when the gross payment is smaller than $\mathrm{DR_N^B}$ then it equals[11] $\mathrm{Y_N}$; see Townsend (1979) and Gale and Hellwig (1985). Hence, the profit function of the bank can be specified as in eq. (2):

[11] Only for the sake of simplicity, here one has assumed that there is only one type of borrower (that remains unknown) and that the collateral c is zero.

$$\Pi_2^B = E_2\left[\min\left(Y_N, DR_N^B\right)\right] - CE_2\left[\frac{\min(Y_N - DR_N^B, 0)}{Y_N - DR_N^B}\right] - R_N D; \tag{6}$$

Note, first of all, that:

$$E_2\left[\min\left(Y_N, DR_N^B\right)\right] = \int_0^{DR_N^B} Y_N f_2(Y_N) dY_N + DR_N^B\left[1 - \int_0^{DR_N^B} f_2(Y_N) dY_N\right] \tag{7}$$

$$E_2\left[\frac{\min\left(Y_N - DR_N^B, 0\right)}{Y_N - DR_N^B}\right] = \int_0^{DR_N^B} \frac{Y_N - DR_N^B}{Y_N - DR_N^B} f_2(Y_N) dY_N + \int_{DR_N^B}^{\infty} \frac{DR_N^B - DR_N^B}{Y_N - DR_N^B} f_2(Y_N) dY_N = \int_0^{DR_N^B} f_2(Y_N) dY_N \tag{8}$$

Assume that the bank, in this context, maximizes its profit with respect to D[12]. So, by the Leibniz rule, one can find the following derivatives:

$$\frac{\partial \int_0^{DR_N^B} f_2(Y_N) dY_N}{\partial D} = R_N^B f_2(DR_N^B) \ ;$$

$$\frac{\partial E_2\left[\min\left(Y_N, DR_N^B\right)\right]}{\partial D} = R_N^B \int_{DR_N^B}^{\infty} f_2(Y_N) dY_N;$$

See Appendix B for further details. Resuming, the first-order conditions are:

$$\frac{\partial \Pi_2^B}{\partial D} = R_N^B \int_{DR_N^B}^{\infty} f_2(Y_N) dY_N - CR_N^B f_2(DR_N^B) - R = 0;$$

$$R_N^B\left[F_2(\infty) - F_2(DR_N^B)\right] - CR_N^B f_2(DR_N^B) - R = 0;$$

$$F_2(DR_N^B) + Cf_2(DR_N^B) = 1 - \frac{R}{R_N^B}; \tag{9}$$

Consider the following Taylor Series approximations of the first order:

$$F_2\left(DR_N^B\right) \cong F_2(E_2[Y_N]) + f_2(E_2[Y_N])\left(DR_N^B - E_2[Y_N]\right);$$

$$f_2\left(DR_N^B\right) \cong f_2(E_2[Y_N]) + f_2{}'(E_2[Y_N])\left(DR_N^B - E_2[Y_N]\right);$$

Substituting them in eq. (9) one obtains:

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[12] This is the variable one chooses in a competitive market.

$$F_2(E_2[Y_N]) + f_2(E_2[Y_N])\left(DR_N^B - E_2[Y_N]\right) + Cf_2(E_2[Y_N]) + Cf_2'(E_2[Y_N])\left(DR_N^B - E_2[Y_N]\right) = 1 - \frac{R}{R_N^B}$$

$$DR_N^B - E_2[Y_N] = \left[\frac{R_N^B - R}{R_N^B} - F_2(E_2[Y_N]) - Cf_2(E_2[Y_N])\right]\frac{1}{f_2(E_2[Y_N]) + Cf_2{\prime}(E_2[Y_N])}$$

$$D = \frac{E_2[Y_N]}{R_N^B} + \frac{1}{(R_N^B)^2}\left[\frac{R_N^B - R - R_N^B[F_2(E_2[Y_N]) + Cf_2(E_2[Y_N])]}{f_2(E_2[Y_N]) + Cf_2{\prime}(E_2[Y_N])}\right]$$

Define $\alpha = [F_2(E_2[Y_N]) + Cf_2(E_2[Y_N])]$; $\beta = f_2(E_2[Y_N]) + Cf_2'(E_2[Y_N])$

$$D^* = \frac{E_2[Y_N]}{R_N^B} + \frac{1-\alpha}{R_N^B\beta} - \frac{R}{(R_N^B)^2\beta} \tag{10}$$

Eq. (10) describes the supply curve of the bank. The possibility of credit rationing can be seen by the fact that D^* is not an increasing function of R_N^B.

In order to find the equilibrium R_N^B in the competitive market, one has just to set the profit function equation to zero. First note that:

$$\Pi_2^B = E_2\left[\min\left(Y_N, DR_N^B\right)\right] - CE_2\left[\frac{\min(Y_N - DR_N^B, 0)}{Y_N - DR_N^B}\right] - R_N D = E_2\left[\min\left(Y_N - C - R_N D, DR_N^B - R_N D\right)\right]$$

$$E_2\left[\min\left(Y_N - C - RD, DR_N^B - R_N D\right)\right] = \int_0^{DR_N^B} Y_N f_2(Y_N)dY_N - (C + R_N D)\int_0^{DR_N^B} f_2(Y_N)dY_N + 1 - \int_0^{DR_N^B} D(R_N^B - R_N)f_2(Y_N)dY_N$$

$$= \int_0^{DR_N^B} Y_N f_2(Y_N)dY_N - \left(C + R_N D + DR_N^B - R_N D\right)\int_0^{DR_N^B} f_2(Y_N)dY_N + 1$$

$$\int_0^{DR_N^B} f_2(Y_N)dY_N = F_2\left(DR_N^B\right) \cong F_2(E_2[Y_N]) + f_2(E_2[Y_N])\left(DR_N^B - E_2[Y_N]\right)$$

$$\int_0^{DR_N^B} Y_N f_2(Y_N)dY_N = \int_0^{\infty} Y_N f_2(Y_N)dY_N - \int_{DR_N^B}^{\infty} Y_N f_2(Y_N)dY_N$$

Since $\int_{DR_N^B}^{\infty} Y_N f_2(Y_N)dY_N = E_2[Y_N]\Phi\left(\frac{\mu+\sigma^2-\ln(DR_N^B)}{\sigma}\right)$ then

$$\int_0^{DR_N^B} Y_N f_2(Y_N)dY_N = E_2[Y_N]\left[1 - \Phi\left(\frac{\mu+\sigma^2-\ln(DR_N^B)}{\sigma}\right)\right] = E_2[Y_N]\Phi\left(\frac{\ln\left(DR_N^B\right)-\mu-\sigma^2}{\sigma}\right)$$

The competitive market condition $\Pi_2^B = 0$ implies:

$$\Pi_2^B = E_2[Y_N]\Phi\left(\frac{\ln(DR_N^B)-\mu-\sigma^2}{\sigma}\right) - \left(C + DR_N^B\right)\left[F_2(E_2[Y_N]) + f_2(E_2[Y_N])\left(DR_N^B - E_2[Y_N]\right)\right] + 1 = 0$$

$$E_2[Y_N]\Phi\left(\frac{\ln(DR_N^B)-\mu-\sigma^2}{\sigma}\right) = \left(C + DR_N^B\right)\left[F_2(E_2[Y_N]) + f_2(E_2[Y_N])\left(DR_N^B - E_2[Y_N]\right)\right] - 1 \qquad (11)$$

On the right side: $\left(C + DR_N^B\right)\left[F_2(E_2[Y_N]) + f_2(E_2[Y_N])\left(DR_N^B - E_2[Y_N]\right)\right] - 1 =$

$$= CF_2(E_2[Y_N]) + DR_N^B F_2(E_2[Y_N]) + CDR_N^B f_2(E_2[Y_N]) + \left(DR_N^B\right)^2 f_2(E_2[Y_N]) \\ - CE_2[Y_N]f_2(E_2[Y_N]) - E_2[Y_N]DR_N^B f_2(E_2[Y_N]) - 1 =$$

$$= \left(DR_N^B\right)^2 f_2(E_2[Y_N]) + DR_N^B[F_2(E_2[Y_N]) + Cf_2(E_2[Y_N]) - E_2[Y_N]f_2(E_2[Y_N])] + CF_2(E_2[Y_N]) \\ - CE_2[Y_N]f_2(E_2[Y_N]) - 1$$

Call $\lambda = F_2(E_2[Y_N]) + f_2(E_2[Y_N])[C - E_2[Y_N]]$; $\gamma = CF_2(E_2[Y_N]) - CE_2[Y_N]f_2(E_2[Y_N]) - 1$

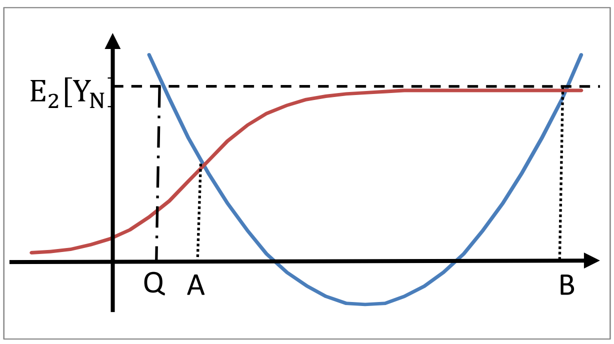
Then, the right-hand side of (11) becomes:

$$\left(DR_N^B\right)^2 f_2(E_2[Y_N]) + DR_N^B\lambda + \gamma \qquad (12)$$

The right-hand side is a parabolic function. Although eq. (11) describes an implicit function of R_N^B, one can consider the following strategy. In order to find viable solutions of the above function one should theoretically find the intersection values between the functions of the right and left sides (points A and B for Graph 3). Indeed, one is only interested in the smallest positive solution as competition should push all the solutions to the smallest one.

However, it is easy to see that point Q is always less than A (as long it is positive), hence one can consider Q as a lower bound of the value A.

Graph 3



Hence, in order to find point Q one has to solve:

$\left(DR_N^B\right)^2 f_2(E_2[Y_N]) + DR_N^B\lambda + \gamma - E_2[Y_N] = 0;$ the smallest solution is:

$$\left(R_N^B\right)_1^* = C = \frac{1}{D}\frac{-\lambda - \sqrt{\lambda^2 - 4f_2(E_2[Y_N])(\gamma - E_2[Y_N])}}{2f_2(E_2[Y_N])} \geq 0 \tag{13}$$

This is the interest rate consistent with the zero profit condition[13]. Combining eq. (10) and eq. (13) one can find the equilibrium interest rate R_N^B only as a function of the underlying parameters and Y_N.

$$\frac{-\lambda-\sqrt{\lambda^2-4f_2(E_2[Y_N])(\gamma-E_2[Y_N])}}{2f_2(E_2[Y_N])} = E_2[Y_N] + \frac{1-\alpha}{\beta} - \frac{R_N}{R_N^B\beta}$$

$$(R_N^B)^* = \frac{R_N}{\beta}\left\{E_2[Y_N] + \frac{1-\alpha}{\beta} - \frac{-\lambda-\sqrt{\lambda^2-4f_2(E_2[Y_N])(\gamma-E_2[Y_N])}}{2f_2(E_2[Y_N])}\right\}^{-1} \geq 0 \tag{14}$$

Eq. (14) also shows the conditions whereby the banking interest rate is higher than the risk-free market interest rate.

The optimal banking behavior when bidding an O-F contract

The profit function, evaluated in period 2, in the case of the option on futures is[14]:

$$E_1[\Pi_2^B] = QCO_1R_2 - \frac{Q}{m}E_1\left[E_2[Y_N]\right] + E_1\left[E_2[Y_N]\frac{Q}{m}\frac{\min(E_2[Y_N],K) - K)}{E_2[Y_N] - K}\right] \tag{15}$$

$$\frac{\partial E_1[\Pi_2^B]}{\partial Q} = CO_1R_2 - \frac{E_1[Y_N]}{m} + \frac{1}{m}E_1\left[E_2[Y_N]\frac{\min(E_2[Y_N],K) - K)}{E_2[Y_N] - K}\right] = 0 \tag{16}$$

---

[13] The condition whereby the solution is real is: $\lambda^2 - 4f(E_2[Y_N])(\gamma - E_2[Y_N]) \geq 0$. In order to satisfy this condition, it is sufficient to show that $\gamma < 0$; $c[F(E_2[Y_N]) - E_2[Y_N]f(E_2[Y_N])] < 1;$ $F(E_2[Y_N]) - E_2[Y_N]f(E_2[Y_N]) < \frac{1}{c};$ $F(E_2[Y_N]) - \frac{1}{c} < E_2[Y_N]f(E_2[Y_N])$.

[14] m is the multiple of the option, such that m>0.

$$\mathrm{CO}_1 = \frac{\mathrm{E}_1[\mathrm{Y}_\mathrm{N}]}{\mathrm{R}_2\mathrm{m}} - \frac{1}{\mathrm{R}_2\mathrm{m}}\mathrm{E}_1\left[\mathrm{E}_2[\mathrm{Y}_\mathrm{N}]\frac{\min(\mathrm{E}_2[\mathrm{Y}_\mathrm{N}],\mathrm{K}) - \mathrm{K})}{\mathrm{E}_2[\mathrm{Y}_\mathrm{N}] - \mathrm{K}}\right] \quad (17)$$

This price equals to both the marginal and the average cost, hence it also ensures zero profits.

The comparison of the costs of the two contracts

Assume that both contracts last for one period only (the T-D contract lasts from period 2 to period 3 and the O-F contract from period 1 to period 2). Note that the comparison of the two contracts should be based on these quantities:

$$(\mathrm{R}_3^\mathrm{B})^* - 1 \lesseqgtr \frac{(CO_1)^* R_2}{W/R_2} - 1 = R_2 - 1 \quad (18)$$

having defined $W = \frac{\mathrm{E}_1[\mathrm{Y}_\mathrm{N}]}{\mathrm{m}} - \frac{1}{\mathrm{m}}\mathrm{E}_1\left[\mathrm{E}_2[\mathrm{Y}_\mathrm{N}]\frac{\min(\mathrm{E}_2[\mathrm{Y}_\mathrm{N}],\mathrm{K})-\mathrm{K})}{\mathrm{E}_2[\mathrm{Y}_\mathrm{N}]-\mathrm{K}}\right]$; the quantities $(CO_1)^*$ and W are multiplied and divided by R_2 (respectively) in order to compare them to the money flows of the left-hand side of (18). Considering eq. (17), the right-hand side is equal to $R_2 - 1$.

Hence, by eq. (18) one can conclude that, in non perverse cases, the cost of the T-D contract is always greater than the O-F contract[15].

However, it is not enough to conclude that the O-F is more efficient since the price is not the only variable that agents may evaluate. In fact, substituting a debt contract with the O-F contract, the agent changes a (costlier) funding of a certain amount with a random funding (the uncertainty is, in fact, internalized). This is true even if the agent has a positive evaluation of his/her future income. A risk averse consumer would not necessarily find it optimal to switch to the O-F contract.

[15] Note that $\mathrm{R}_3^\mathrm{B} > \mathrm{R}_3$ because of the riskiness of the banking loan; hence the T-D contract is always more expensive than the O-F contract in the case of a flat term structure of interest rate ($\mathrm{R}_2 = \mathrm{R}_3$).

The optimal choice of a non risk-neutral consumer in the case of adverse selection and moral hazard.

The consumer maximizes his utility function by choosing the composition of a portfolio of T-D and O-F whose share of T-D is x and the share of O-F is (1-x). In this way, one shows the condition whereby the consumer chooses either contract. Assume, without loss of generality, that N=3 and that the collateral c is zero but there are verification costs C.

For the sake of exposition, define:

$$\mathrm{min} = \min(\mathrm{Y}_3, \mathrm{DR}_3^{\mathrm{B}}) + \mathrm{C}\frac{\min(\mathrm{Y}_3-\mathrm{DR}_3^{\mathrm{B}},0)}{\mathrm{Y}_3-\mathrm{DR}_3^{\mathrm{B}}} \tag{19}$$

$$\mathrm{max} = \frac{\mathrm{Q}}{\mathrm{m}}\max(\mathrm{E}_2[\mathrm{Y}_3]/\mathrm{R}_3 - \mathrm{K}, 0) \tag{20}$$

Utility and budget constraints are then (where U(.) is a well-behaved concave utility function, β is a discount factor and C_i is the consumption stream):

$$\mathrm{U} = \mathrm{E}_1\left[\sum_{i=1}^{3} \beta^{i-1}\,\mathrm{U}(\mathrm{C_i})\right] \text{ s.t.:} \tag{21}$$

$$W_2 = (Y_1 + W_1 - C_1 - (1-x)CO_1Q)R_2; \tag{22}$$

$$W_3 = (Y_2 + W_2 - C_2 + xD + (1-x)\mathrm{max})R_3 \geq 0; \tag{23}$$

where W_1 is the initial wealth. The inequality constraint stems from the fact that the first factor (the saving in period 2) cannot be negative because otherwise the agent would be allowed to borrow at the rate R_3.

$$C_3 = W_3 + Y_3 - x\mathrm{min};$$

$$W_3 = (Y_2 + (Y_1 + W_1 - C_1 - (1-x)CO_1Q)R_2 - C_2 + xD + (1-x)\mathrm{max})R_3;$$

$$W_3 = (Y_2 + (Y_1 + W_1 - C_1)R_2 - C_2)R_3 + (xD - (1-x)CO_1QR_2 + (1-x)\mathrm{max})R_3;$$

Define $H_2 = Y_2 + (Y_1 + W_1 - C_1)R_2 - C_2)R_3$

$$C_3 = H_2 + (xD - (1-x)CO_1QR_2 + (1-x)\mathrm{max})R_3 - x\mathrm{min} + Y_3; \tag{24}$$

For the sake of exposition, one substitutes C_3 through the budget constraint of eq. (24) and then one sets up the Lagragian where the multiplier of inequality constraint of eq. (23) is set explicitly[16]. The derivative of this Lagrangian with respect to x is:

$\frac{dL}{dx} = \frac{d\beta^2 \mathrm{E}_2[\mathrm{U}(\mathrm{C}_3)]}{dx} = -\mu$; where μ is the Lagrange multiplier of the constraint of eq. (26) with $\mu \geq 0$. This implies :

$$\mathrm{E}_2[\mathrm{U}'(\mathrm{C}_3)\{(CO_1QR_2R_3 - \max + \mathrm{D})R_3 - \min\}] = -\mu; \qquad (25)$$

$$\mathrm{E}_2[\mathrm{U}'(\mathrm{C}_3)(\mathrm{D}R_3 - \min)] = \mathrm{E}_2[\mathrm{U}'(\mathrm{C}_3)(\max - CO_1QR_2R_3)] - \mu; \qquad (26)$$

Disregard for now the term μ . Eq. (26) shows that the optimal solution x^* must equal the marginal net benefits of both contracts. Consider now a non-equilibrium situation; if, for example, the left-hand side would be larger than the right-hand side, it would be optimal to increase the share of the T-D and hence to increase x. Consequently, one has to find the condition whereby the right-hand side is always larger than the left-hand side so that the optimal solution is $\mathrm{x}^* = 0$, namely, only the O-F contract is chosen. Also note that since x is chosen in period 1, the expected value of period 1 must be applied to both sides of equations. That is:

$$\mathrm{E}_1[\mathrm{U}'(\mathrm{C}_3)(\max - CO_1QR_2R_3)] > \mathrm{E}_1[\mathrm{U}'(\mathrm{C}_3)(\mathrm{D}R_3 - \min)] - \mu; \qquad (27)$$

$$\mathrm{E}_1[\mathrm{U}'(\mathrm{C}_3)\max] - \mathrm{E}_1[\mathrm{U}'(\mathrm{C}_3)]CO_1QR_2R_3 > \mathrm{E}_1[\mathrm{U}'(\mathrm{C}_3)]\mathrm{D}R_3 - \mathrm{E}_1[\mathrm{U}'(\mathrm{C}_3)\min] - \mu;$$

$$\mathrm{cov}_1[\mathrm{U}'(\mathrm{C}_3)\max] + \mathrm{E}_1[\mathrm{U}'(\mathrm{C}_3)]\mathrm{E}_1[\max] - \mathrm{E}_1[\mathrm{U}'(\mathrm{C}_3)]CO_1QR_2R_3 > \mathrm{E}_1[\mathrm{U}'(\mathrm{C}_3)]\mathrm{D}R_3 - \mathrm{cov}_1[\mathrm{U}'(\mathrm{C}_3)\min] - \mathrm{E}_1[\mathrm{U}'(\mathrm{C}_3)]\mathrm{E}_1[\min]; \qquad (28)$$

After rearranging and noting that[17]

$$\mathrm{cov}_1[\mathrm{U}'(\mathrm{C}_3), \min] \cong \mathrm{E}_1[\mathrm{U}''(\mathrm{C}_3)]\mathrm{cov}_2[\mathrm{C}_3, \min] \qquad (29)$$

$$\mathrm{cov}_1[\mathrm{U}'(\mathrm{C}_3), \max] \cong \mathrm{E}_1[\mathrm{U}''(\mathrm{C}_3)]\mathrm{cov}_2[\mathrm{C}_3, \max] \qquad (30)$$

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[16] For an introduction to the Kuhn-Tucker conditions for a maximization with an inequality constraint see Varian, 1992.

[17] This approximation is obtained by the Taylor-series expansion; since one is only interested in showing the inequality, this approximation serves only for clarification purposes.

$$\mathrm{E}_1[\mathrm{U}''(\mathrm{C}_3)]\mathrm{cov}_1[\mathrm{C}_3, \max] + \mathrm{E}_1[\mathrm{U}'(\mathrm{C}_3)](\mathrm{E}_1[\max] - CO_1QR_2R_3) > \mathrm{E}_1[\mathrm{U}'(\mathrm{C}_3)](\mathrm{D}R_3 - \mathrm{E}_1[\min]) - \mathrm{E}_1[\mathrm{U}''(\mathrm{C}_3)]\mathrm{cov}_1[\mathrm{C}_3, \min] - \mu; \quad (31)$$

Alternatively, one could specify a process for "min" and "max" in order to find the relevant moments as in Cochrane 2001. Assume now that this representative consumer has good expectations about Y_3. More precisely assume that, $\forall$ Z sufficiently large, it holds:

$$\frac{\mathrm{E}_1[\mathrm{Y}_3]/\mathrm{R}_3 - \mathrm{K}}{\mathrm{var}_1[\mathrm{Y}_3]} > Z \quad (32)$$

This means that:

$$\max = \frac{\mathrm{Q}}{\mathrm{m}} \max(\mathrm{E}_2[\mathrm{Y}_3]/\mathrm{R}_3 - \mathrm{K}, 0) \cong \frac{\mathrm{Q}}{\mathrm{m}}(\mathrm{E}_2[\mathrm{Y}_3]/\mathrm{R}_3 - \mathrm{K}) \quad (33)$$

$$\min = \min(\mathrm{Y}_3, \mathrm{DR}_3^{\mathrm{B}}) + \mathrm{c}\frac{\min(\mathrm{Y}_3 - \mathrm{DR}_3^{\mathrm{B}}, 0)}{\mathrm{Y}_3 - \mathrm{DR}_3^{\mathrm{B}}} \cong \mathrm{DR}_3^{\mathrm{B}} \quad (34)$$

Eq. (34) then becomes:

$$\mathrm{E}_1[\mathrm{U}''(\mathrm{C}_3)]\mathrm{cov}_1[\mathrm{C}_3, \frac{\mathrm{Q}}{\mathrm{m}}\mathrm{E}_2[\mathrm{Y}_3]/\mathrm{R}_3] + \mathrm{E}_1[\mathrm{U}'(\mathrm{C}_3)](\mathrm{E}_1\left[\frac{\mathrm{Q}}{\mathrm{m}}(\mathrm{E}_2[\mathrm{Y}_3]/\mathrm{R}_3 - \mathrm{K})\right] - CO_1QR_2R_3) > \mathrm{E}_1[\mathrm{U}'(\mathrm{C}_3)](\mathrm{D}R_3 - \mathrm{E}_1[\mathrm{DR}_3^{\mathrm{B}}]) - \mathrm{E}_1[\mathrm{U}''(\mathrm{C}_3)]\mathrm{cov}_1[\mathrm{C}_3, \mathrm{DR}_3^{\mathrm{B}}] - \mu; \quad (35)$$

noting that $\mathrm{cov}_1[\mathrm{C}_3, \mathrm{DR}_3^{\mathrm{B}}] = 0$ and that,

$$\begin{aligned}\mathrm{cov}_1[\mathrm{C}_3, \mathrm{E}_2[\mathrm{Y}_3]] = \mathrm{cov}_1[\mathrm{Y}_3, \mathrm{E}_2[\mathrm{Y}_3]] &= \mathrm{E}_1[(\mathrm{Y}_3 - \mathrm{E}_1[\mathrm{Y}_3])(\mathrm{E}_2[\mathrm{Y}_3] - \mathrm{E}_1[\mathrm{E}_2[\mathrm{Y}_3]]) \\ &= \mathrm{E}_1[(\mathrm{Y}_3 - \mathrm{E}_1[\mathrm{Y}_3])(\mathrm{E}_2[\mathrm{Y}_3] - \mathrm{E}_1[\mathrm{Y}_3]) \\ &= \mathrm{E}_1[\mathrm{Y}_3\mathrm{E}_2[\mathrm{Y}_3] - \mathrm{Y}_3\mathrm{E}_1[\mathrm{Y}_3] + \mathrm{E}_1[\mathrm{Y}_3]\mathrm{E}_2[\mathrm{Y}_3] - \mathrm{E}_1[\mathrm{Y}_3]\mathrm{E}_1[\mathrm{Y}_3]] = 0\end{aligned}$$

having assumed that the only correlation between C_3 and $\mathrm{E}_2[\mathrm{Y}_3]$ works through Y_3 itself. Simplifying further:

$$(\mathrm{Q}/\mathrm{m})(\mathrm{E}_1[\mathrm{Y}_3]/\mathrm{R}_3 - K) - CO_1QR_2R_3 > DR_3 - \mathrm{DR}_3^{\mathrm{B}} - \mu; \quad (36)$$

Since the right-hand side is always negative[18], eq. (36) becomes

$$\frac{1}{\mathrm{m}}(\mathrm{E}_1[\mathrm{Y}_3]/\mathrm{R}_3 - K) > CO_1 R_2 R_3 \quad (37)$$

Note that this condition is always satisfied provided that the agent is long about his or her future income: the expected pay-off of the O-F is larger than its opportunity cost. This implies that a "favorable" selection occurs that causes a "screening device" between consumers. Recall that this screening device, when the bank offers the T-D contract, worsens the expected profit of the bank aggregated on the pool of the consumers. In fact, $H(\theta)$ changes since consumers who would like a T-D (they have insufficient positive expectations on Y_3, in the sense defined above) cause a decline in the expected revenue $\mathrm{E}_2\left[\min\left(\mathrm{Y}_\mathrm{N}, \mathrm{DR}_\mathrm{N}^\mathrm{B}\right)\right]$.

In other words, when this screening device is applied, one is able to state that the T-D contract between the bank and the consumer with "good expectations" is Pareto inefficient, because:

- the bank is indifferent about offering either contract since both contracts are constructed such that they ensure the zero-profit condition;
- the consumer with good expectations (as defined above) on the future income always prefers the O-F contract.

It follows that two separate equilibriums are possible:

- in the O-F market, where assumptions of the welfare theorems apply;
- in the T-D market, where the bank either can charge a higher interest rate $\mathrm{R}_\mathrm{N}^\mathrm{B}$ conditional on the consumers' demand curve or it finds it not optimal to stay in this market as its profit remains a not monotonically increasing function of $\mathrm{R}_\mathrm{N}^\mathrm{B}$.

Note that risk of bankruptcy has been internalized in the profit functions of the borrowers through the O-F contract. In this perspective, the O-F is a solution of a negative externality created by borrowers with hidden "bad" expectations.

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[18] This is because $\mathrm{R}_3^\mathrm{B} > \mathrm{R}_3$ and because $\mu \geq 0$. Indeed, one can show that $\mu > 0$ (strictly) because otherwise the inequality constraint would not be binding (see Varian, 1992); this, in turn, is not optimal because the agent would borrow at the rate $\mathrm{R}_3^\mathrm{B}$and invest at the rate R_3.

Conclusions

Throughout the paper, one has shown that an option on futures may solve the market inefficiency (or failure) due to asymmetry of information among the market participants. It has been explained that the fundamental reason for this being possible is that borrowers who buy this option must be "long" in the sense that they have enough good expectations of their future profit (correlated with the payoff of the option). This means that (rational) "dishonest" borrowers will not use their private information "against themselves". Another way to see this is that, under asymmetric information, the selection between potential borrowers is "adverse" in the sense that after the selection only borrowers with "bad" expectations of their profits remain willing to obtain the funds; this selection works through the costs' side of the borrower´s profit function. With the option on futures contract, the selection is "favorable" because it works through the revenue's side of the borrower´s profit function. This also means that the risk of bankruptcy in models with adverse selection and moral hazard are basically shifted from the borrower to the lender; conversely, with the option on futures contract, this risk is "re-internalised" in the profit function of the lender. In other words, with asymmetric information, there are negative externalities in the credit market and hence the market becomes Pareto inefficient, which is not the case with the option on futures market.

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Appendix A:

Since Y_N is logarithmically distributed, then $E_2[Y_N] = e^{\mu+\frac{1}{2}\sigma^2}$. Fixing $E_2[Y_N] = \overline{E_2[Y_N]}$ one obtains $\mu = \ln\overline{E_2[Y_N]} - \frac{1}{2}\sigma^2$. At the same time, it comes directly from the definition of the variance that : $Var_2[Y_N] = \left(\overline{E_2[Y_N]}\right)^2(e^{\sigma^2} - 1)$; this means that an increase in $Var_2[Y_N]$ implies an increase in σ^2 which implies a decrease in μ. Now note that the median is e^{μ}. The increase in $Var_2[Y_N]$ implies that the probability mass of Y_N concentrates leftwards since the median is also decreased. Hence the probability that $Y_N < DR_N^B$ is increased. This implies that $\int_0^{DR_N^B} Y_N f_2(Y_N)dY_N = E_2\left[Y_N | Y_N < DR_N^B\right]\Pr[Y_N < DR_N^B]$ increases and $DR_N^B \int_{DR_N^B}^{\infty} f_2(Y_N)dY_N$ decreases. By definition, all this implies a decrease in $E_2\left[\min\left(Y_N, DR_N^B\right)\right]$.

Appendix B:

$$\frac{\partial \int_0^{DR_N^B} Y_N f_2(Y_N)dY_N}{\partial D} = R_N^B DR_N^B\, f_2(R_N^B) - 0 + \int_0^{DR_N^B} 0\, f_2(Y_N)dY_N\ ;$$

$$\frac{\partial \int_{DR_N^B}^{\infty} DR_N^B f_2(Y_N)dY_N}{\partial D} = 0 - R_N^B DR_N^B f_2(R_N^B) + R_N^B \int_0^{DR_N^B} f_2(Y_N)dY_N\ .$$

The derivation of the above integrals were performed trough the Leibniz rule. The conditions whereby this rule can be applied are:

- the function under the integral sign is continuous along with its first derivative;
- the functions of the endpoints are continuous and derivable.

These conditions are always satisfied. Furthermore, the latter integral is defined as improper since the upper endpoint is ∞. This implies that one should also verify its "uniform convergence" in order to apply the Leibniz rule. To this purpose, it is sufficient to show that $\int_{DR_N^B}^{\infty} DR_N^B f_2(Y_N) dY_N < \infty$ which is very easy to prove.