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Abstract

We present new empirical evidence on the distribution of earnings, income and wealth among entrepreneurs in Germany. We document that both earnings and income are more concentrated among entrepreneurs than among workers and describe a large-scale overlapping-generations model that can replicate these findings. As an application, we compute the equilibrium effects of a reform of the German pay-as-you-go pension system in which entrepreneurs must also contribute and receive a pension. We show that in the presence of mobility between workers and entrepreneurs, the expected lifetime utility of all newborn households unanimously declines due to the general equilibrium effects of lower aggregate savings, and welfare losses amount to approximately 5% of total consumption. In addition, the integration of self-employed workers into the social security system in Germany does not help to improve its fiscal sustainability, and only an increase in the retirement age to 70 years will help to finance pensions at the present level beyond the year 2050.

JEL classification: H55, D31, D15, J11, L26, C68

Keywords: Entrepreneurship, aging, income distribution, overlapping generations, social security, fiscal sustainability

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1. Introduction

Worldwide, pension systems are being continually reformed. The main cause of stress on pension systems is the global demographic shift towards higher longevity and lower fertility. To counter the effects of the rising dependency ratio, several policies are available. In the recent past, many countries in the OECD have decided to raise the retirement age – on average, the normal retirement age will increase by almost two years by approximately 2060 (OECD, 2019, p. 17). Other policy options include changes in contribution rates during working life or replacement rates of pensioners. Another major policy measure regards extending the groups who are subject to mandatory contributions, such as the informal sector or entrepreneurs. Across the OECD, voluntary or mandatory access to pension plans varies widely for entrepreneurs and non-standard categories of workers.

Including entrepreneurs in the public pension system is attractive, as they would contribute immediately to the pension system, thus alleviating the current pension burden. In the long run, however, this gain must be traded off against the future entitlements of these entrepreneurs. There are also arguments in favor of exempting entrepreneurs from social security contributions: compared to regular employees, they have a higher degree of discretion in calculating their contribution base, which might be considered unfair. Furthermore, not being required to pay into the public pension system might induce more individuals to start their own businesses. In this way, economic policy can promote entrepreneurship, and more businesses will mean higher total employment. In this paper, we explicitly model the age-dependent share of entrepreneurs in each cohort in a large-scale overlapping-generations (OLG) model calibrated to the Germany economy. The calibrated model is then used to run an experiment. We investigate the consequences of extending the German PAYG pension system to entrepreneurs and find that the general equilibrium effects are harmful to both entrepreneurs and workers.

This paper contributes to various strands of literature. First, it provides empirical evidence regarding the life-cycle behavior of entrepreneurs' earnings, income and wealth. Many empirical studies do not differentiate between entrepreneurs and workers or focus exclusively on the earnings or income of male workers (female workers are often disregarded, as their employment history tends to be more fragmented than that of males, at least for earlier cohorts). For instance, Guvenen (2009) investigates the risk structure of the labor incomes of men without distinguishing between workers and entrepreneurs. He concludes that heterogeneity in the deterministic component of income paths should be taken into account. Guvenen et al. (2015) offers a detailed statistical analysis of earnings changes using data on male employees. The study shows that earnings shocks are nonnormally distributed and have substantial skewness and kurtosis.

A comprehensive overview of the distribution of earnings, income and wealth in the U.S. is given by Budría Rodríguez et al. (2002). They confirm the finding of earlier studies that wealth is by far the most unequally distributed variable, and earnings is less equally distributed than income for the main part of the distribution. Looking at the correlation between earnings and wealth, Budría Rodríguez et al. (2002) report relatively small values (0.47); the correlation between income and wealth (0.60) is also lower than the correlation between earnings and income (0.72). Furthermore, they find that employment status, i.e., whether an individual is a worker, self-employed, retired or a non-worker, is a crucial variable: workers tend to be wealth-poor, whereas retirees have higher wealth on average. The self-employed have a much higher share of income and wealth than their population share.

Quadrini (2000) provides a comparative analysis of the differences in income and wealth between workers and entrepreneurs in the U.S. He finds that there is a high share of entrepreneurs in the top wealth group. Considering the distinction between entrepreneurs and workers is therefore critically important to understanding this strong wealth concentration. Quadrini (2000) further shows that entrepreneurs have a higher wealth-to-income ratio than workers. Bhandari et al. (2020) examine the reliability of several U.S. survey datasets on business income and wealth. Their analysis contains all types of business, i.e., it also includes corporations. According to Bhandari et al. (2020), the evidence suggests that survey data "should be treated with great caution", as they create considerable measurement problems, particularly concerning business valuations. Using data from the German Socioeconomic Panel, Grabka and Westermeier (2014) describe the wealth distribution in Germany. Even though the definition of wealth is rather comprehensive in that survey, public pension entitlements originating from the social security system are neglected. Another source of measurement problems is the absence of individuals with extremely high wealth in the survey. The Gini coefficient of net wealth in Germany is higher than that in any other country in the Euro zone. Computing the life-cycle profiles of wealth, Grabka and Westermeier (2014) document an inverse-U shaped development with net wealth peaking around retirement age. Entrepreneurs are found to have the highest wealth level; in particular, they are more likely to have assets in the form of private insurance or business assets.

Second, our policy analysis is related to the extensive literature on the implications of aging for the sustainability of social security systems based on multi-period overlapping-generations models. As one of the earliest and most prominent studies on the consequences of the effects of demographic transition on public pensions, De Nardi et al. (1999) evaluate various policy instruments in a dynamic general equilibrium model with overlapping generations. They advocate a switch to a purely defined contributions system. Nishiyama and Smetters (2007) analyze a 50% privatization of social security and find the welfare effects to be sensitive to the assumptions of a closed economy, missing annuities markets, and the progressivity of pensions. In a more recent study, Kitao (2014) finds that reducing pension benefits is the most efficient policy in the long run. Heer and Irmen (2014) also consider the effect of pension reform on the endogenous growth rate. If labor supply becomes scarcer due to aging, firms have a higher incentive to invest in labor-augmenting technological change. The growth rate effect is shown to be largest for the case of a frozen contribution rate so that the level of pensions falls in comparison to policies with i) a constant pension level or ii) a higher retirement rate. Heer et al. (2020)

study the sustainability of PAYG public pension systems in the U.S. and in 14 European countries. For the present pension systems in these countries, as characterized by the replacement rate of pensions with respect to wage income and the effective retirement age, they find that the majority of continental European countries, including France, Italy, Spain, and Germany, cannot finance pensions beyond the year 2040 through a social security tax on labor income alone. In contrast, the English-speaking countries – the U.S., the UK and Ireland – have sufficient fiscal space to finance future increases in public pension expenditures because their present tax rates on labor income are still a large distance away from the top of the Laffer curve. We add three results to this literature.[1] 1) We demonstrate that Germany will be unable to finance the PAYG pension system in the year 2050 through taxes on labor income alone, even if entrepreneurs contribute to the public pension system as well. 2) We show that an increase in the (effective) retirement age to 70 helps to establish the sustainability of the public pension system. 3) Both under the present demographics and the population parameters prevailing in the year 2050, the average welfare (as measured by expected lifetime utility) of newborn households decreases if entrepreneurs also contribute to the public pension system in Germany.

Third, a variety of studies analyze the optimal amount and schedule of public pensions in the steady state, including İmrohoroğlu et al. (1995), İmrohoroğlu et al. (1999), and Fehr et al. (2013). They find that the optimal replacement rate of pensions relative to gross wages is rather low and should not exceed 20%. Fehr et al. (2013) also consider the optimal progressivity of pensions in Germany and find that the earnings-related part should be small so that the pension system substantially redistributes among retirees. Our model adds to this literature by considering a broadening of the contribution base. In addition, we provide a sensitivity analysis of the latter result of Fehr et al. (2013) by

[1] Our discussion of the literature on the sustainability of public pensions in light of the demographic transition is by no means exhaustive. Other articles that have considered demographic transitions and their effects on the pension system include Braun and Joines (2015), Conesa and Garriga (2016), İmrohoroğlu and Kitao (2009), İmrohoroğlu et al. (2016), Krueger and Ludwig (2007), among others.

showing that the pension system in Germany should also be more progressive if we add entrepreneurship to the model and allow entrepreneurs to contribute to the PAYG system. A lump-sum pension to all retirees is demonstrated to imply higher welfare than a purely contributions-based pension (which is the prevailing pension system in Germany).

This paper is organized as follows. In Section 2, we describe the data from Germany and present age-specific statistics for both workers and entrepreneurs together with inequality measures. Section 3 presents a simple model without mobility that helps to understand the basic life-cycle profiles of workers and entrepreneurs with different productivity levels and allows us to calibrate the productivity of entrepreneurs in self-employment. In Section 4, we describe our results and replicate some of the empirical observations on the distribution of income and wealth. In addition, we show that the expected lifetime utility of low earners among workers is higher than that of the same group among entrepreneurs, while the opposite holds for the high earners. This phenomenon is also reflected in higher inequality among entrepreneurs both empirically and in our model. Section 5 considers a policy experiment in which entrepreneurs also contribute to the German pay-as-you-go pension system. In Section 6, we extend our benchmark model to include some more realistic features of the pension and income tax system in Germany, e.g., progressive income taxes and contributions-based pensions. In addition, we allow for mobility so that workers may become entrepreneurs and vice versa or switch productivity types. Section 7 concludes. The Appendix describes the large-scale OLG model and the computational methods in more detail.

2. Earnings, income and wealth distribution among workers and entrepreneurs

This section describes the main features of the earnings, income and wealth distribution of entrepreneurs and workers in Germany.

2.1. Data description

Our data source is the German Social-Economic Panel dataset[2] (SOEP) provided by the German Institute for Economic Research/DIW Berlin (Goebel et al., 2018). We make use of the Cross National Equivalent File (CNEF) and the wealth samples, which are part of the SOEP data distribution (Grabka, 2020). The CNEF earnings variable consists of wages and salaries, including training, secondary jobs, bonuses, over-time compensation and profit-sharing, as well as earnings from self-employment, defined as profits before taxes. Gross earnings from self-employment are also reported as a separate variable. The total population consists of all individuals regardless of whether they are economically active. We restrict our analysis to the economically active population consisting of individuals (i) aged between 20 and 80, (ii) working between 950 and 4000 hours per year ("full-time"), and (iii) having strictly positive earnings either from dependent work – workers – or from self-employment – entrepreneurs. In our analysis, entrepreneurs are defined as individuals whose earnings from self-employment constitute at least 75% of their total earnings. For entrepreneurs, earnings from sources other than self-employment are disregarded. Years before 1995 have been dropped so that we can consider both West and East Germany and avoid distortions due to drastic changes in the first few years after reunification. All observations are weighted using either cross-sectional weights or, for issues regarding two periods, longitudinal weights.

Information on gross and net wealth is available only for the years 2002, 2007, 2012 and 2017. Total gross wealth includes housing (ownership of a home and other property), financial assets, building-loan contracts, private insurance, business assets and tangibles. Net wealth is computed as gross wealth minus total debts (mortgages, consumer credits and other debts). The SOEP provides multiple imputations for the wealth variables with item non-response (Grabka, 2015). As we are not primarily concerned with the variation in the estimates,

[2]Socio-Economic Panel (SOEP), data for years 1984-2018, version 35, 2020, doi: 10.5684/soep.v35.

we take into account only one of the imputations.

The income and wealth variables have been inflation adjusted using annual CPI values published by the Federal Statistical Office (Statistisches Bundesamt, 2020). Table 1 reports descriptive statistics when observations are pooled over all years (1995-2018). Fig. 1 shows the proportion of entrepreneurs per year in

	Mean	Standard deviation
Age	43.0	11.3
Hours worked	2104.7	528.3
Earnings	36973.5	28907.8
Pre-government income	60874.1	47281.2
Post-government income	44041.4	28272.3
Gross wealth	133186.7	474993.4
Net wealth	105776.0	436893.4

Table 1: Descriptive statistics: Mean and standard deviation of pooled observations. Hours worked, earnings, pre- and post-government incomes are annual quantities; earnings, incomes and wealth are inflation adjusted (in 2015 Euros).

the sample. The share increased in the first half of the observation period and slightly decreased since then. Its long-run average is approximately 8%.

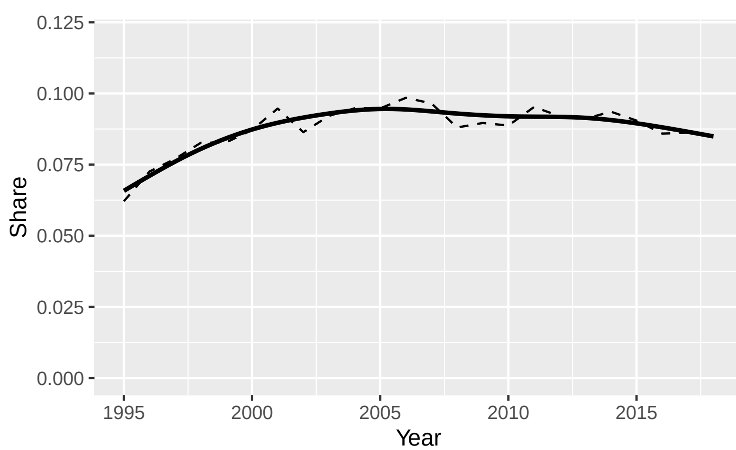

Figure 1: Proportion of entrepreneurs (dashed line); the solid line is the smoothed time series.

2.2. Earnings over time

For both groups (workers and entrepreneurs), we identify individuals as highly productive in year t if their hourly earnings are above the median hourly earnings of their group in year t. Both workers and entrepreneurs can jump between productivity levels from year to year. The mean real hourly earnings of workers and entrepreneurs of both productivity levels are shown by the dashed lines in Fig. 2. The solid lines are the corresponding smoothing splines. The smoothed overall means (for workers and entrepreneurs of both productivity levels) are shown by the dotted lines in the middle. In any year, low-productivity workers earn more per hour than low-productivity entrepreneurs. For highly productive individuals, the relation reverses: the overall mean earnings per hour are larger for entrepreneurs. The spread between high-earnings workers and entrepreneurs increases over time.

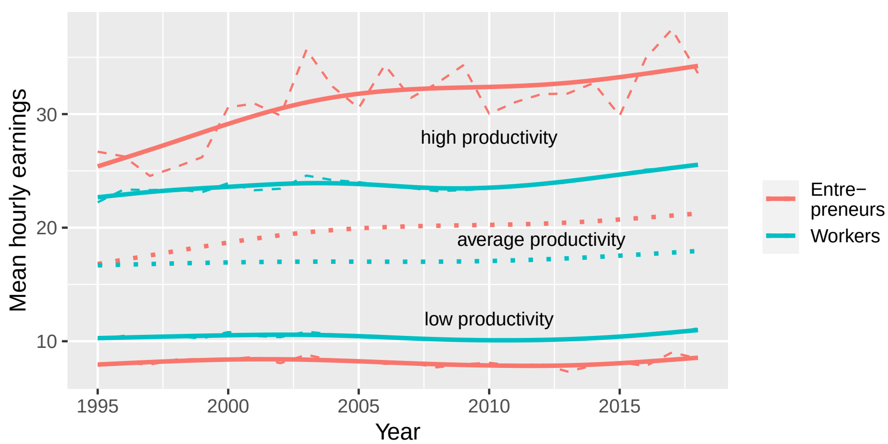

Figure 2: Mean hourly earnings for entrepreneurs and workers with low or high productivity levels (dashed lines) and the smoothed time series (solid lines); the dotted lines are the overall productivity mean hourly earnings of both groups.

2.3. Age profiles

The age profile of the share of entrepreneurs is depicted in Fig. 3. Individuals are binned into five-year age groups (20-24, 25-29, . . . , 75-80) to obtain more stable estimates. The colored lines show the age profiles for selected years. Evidently, the lines shift to the right over time, at least for the older age groups. The rising share, especially for individuals aged 45+, is likely to have been caused by labor market policies starting in the 1990s that encouraged the unemployed to move into self-employment (Caliendo et al., 2016). Averaging over time, the share of entrepreneurs is roughly 1% for 21-year-olds.

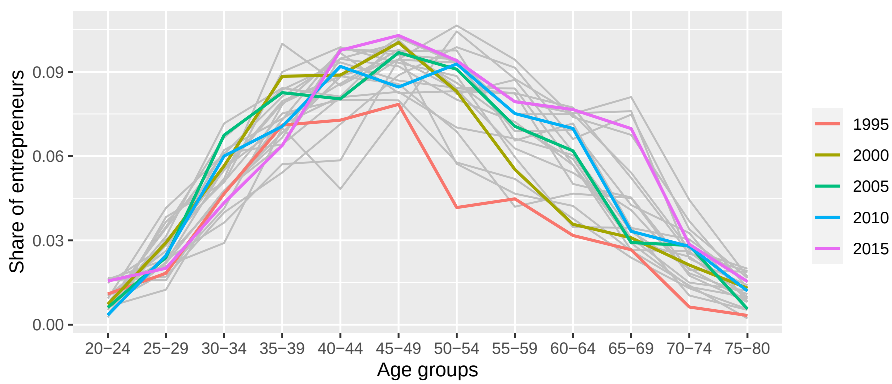

Figure 3: Age profile of the proportion of entrepreneurs in the total population. Observations are binned into five-year age groups.

Mean hourly earnings also depend on age. Fig. 4 shows the age profiles of mean real hourly earnings for workers and entrepreneurs of both productivity levels, where the observations have been pooled over all years. With the exception of low-productivity workers, the profiles decline for very young ages. For highly productive individuals (workers or entrepreneurs), the earnings profile then rises until retirement. In contrast, workers and entrepreneurs of the low-productivity type experience falling wages after approximately 50 years of age. High-productivity entrepreneurs earn higher hourly wages than high-productivity workers. In contrast, low-productivity entrepreneurs earn less than low-productivity workers.

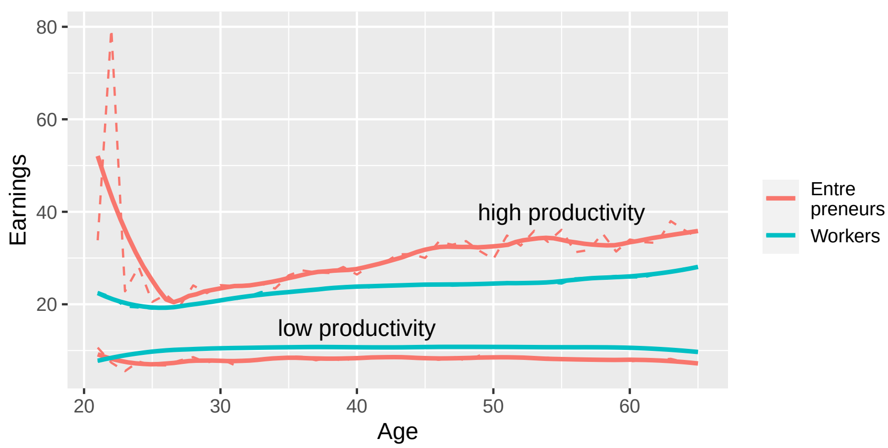

Figure 4: Age profile of the mean real hourly earnings of entrepreneurs and workers of both productivity levels. The solid lines are the smoothed profiles.

2.4. Inequality of income and wealth among workers and entrepreneurs

We proceed to describe the evolution of economic inequality. Fig. 5 shows the Gini coefficients of earnings, gross income and net income[3] for workers and entrepreneurs. The level of inequality is considerably higher for entrepreneurs, while the change over time is roughly similar in both groups. Apart from random fluctuations, inequality did not change much before 2000 and then started to increase markedly for approximately 10 years. Since 2010, inequality has been relatively stable. For both groups, earnings are more unequally distributed than pre-government income. Due to the progressive tax and benefit system, post-government income is the most equally distributed variable. Fuchs-Schündeln et al. (2010) decompose earnings inequality but find that a large share cannot be explained by the observable variables.

Regarding wealth inequality, Table 2 reports the Gini coefficients of gross and net wealth for the years 2002, 2007, 2012, and 2017. Consistent with findings in other studies (Grabka and Westermeier, 2014), wealth inequality is extremely

[3]Gross income is identified by the CNEF variable for pre-government income, and net income is defined by the CNEF variable for post-government income (Grabka, 2020).

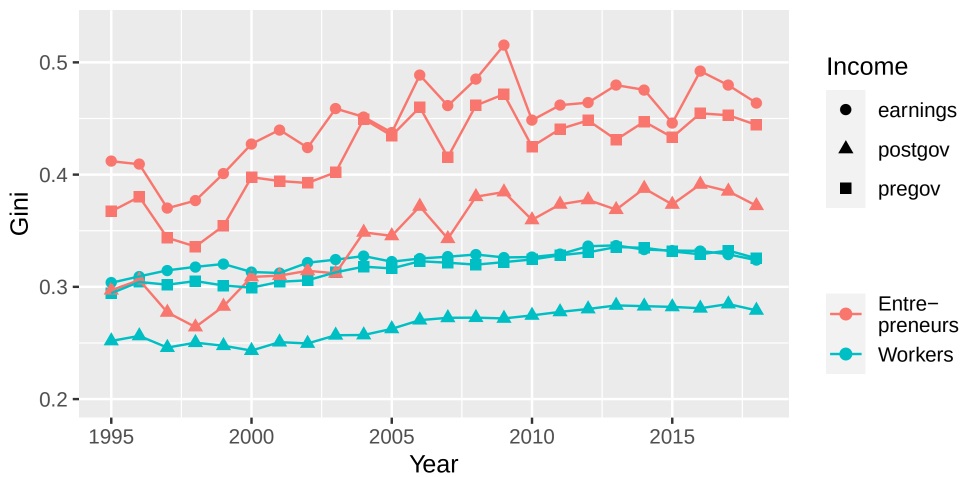

Figure 5: Gini coefficients of the earnings, gross income and net income of entrepreneurs and workers.

high. Net wealth inequality is higher than gross wealth inequality, indicating that individuals with low gross wealth tend to have higher debt than richer individuals.

	All		Workers		Entrepreneurs	
Year	gross wealth	net wealth	gross wealth	net wealth	gross wealth	net wealth
2002	0.7152	0.7738	0.6983	0.7589	0.6447	0.6923
2007	0.7233	0.7932	0.6835	0.7617	0.7322	0.7825
2012	0.7106	0.7782	0.6780	0.7565	0.7232	0.7491
2017	0.7214	0.7765	0.6826	0.7478	0.7332	0.7521

Table 2: Gini coefficients of gross and net wealth.

The correlation between earnings and wealth is relatively small (see Table 3). This finding is in line with other studies, e.g., Budría Rodríguez et al. (2002). The correlation estimates are rather volatile over time, and there is no clear pattern for the relation between workers and entrepreneurs. The estimates become more stable when we compute rank correlations (not reported), but again, there is no clear distinction between workers and entrepreneurs.

	All		Workers		Entrepreneurs	
	gross	net	gross	net	gross	net
Year	wealth	wealth	wealth	wealth	wealth	wealth
2002	0.2409	0.2266	0.2181	0.2127	0.2318	0.2042
2007	0.2870	0.2556	0.3014	0.2612	0.2803	0.2500
2012	0.3550	0.3381	0.3670	0.3440	0.4973	0.4671
2017	0.2776	0.2574	0.3635	0.3462	0.2771	0.2575

Table 3: Correlation coefficients of gross wealth with earnings and of net wealth with earnings.

2.5. Mobility

Individuals can experience two kinds of transitions: they can jump between productivity levels, and they can change from being a worker to being an entrepreneur or vice versa. Assuming time-invariant transition probabilities, the productivity level transition matrix for workers is estimated as

$$P_w = \begin{bmatrix} 0.8554 & 0.1446 \\ 0.1450 & 0.8550 \end{bmatrix}.$$

For example, the probability for a worker to jump from low productivity in one year to high productivity in the next year is 14.46%. The corresponding transition matrix for entrepreneurs is

$$P_e = \begin{bmatrix} 0.7847 & 0.2153 \\ 0.1994 & 0.8006 \end{bmatrix}.$$

Individuals can also change their group (worker/entrepreneur). Assuming that these transition probabilities are constant over time, the transition matrix for changes from worker to entrepreneur or vice versa is

$$P_{w\leftrightarrow e} = \begin{bmatrix} 0.9929 & 0.0071 \\ 0.0940 & 0.9060 \end{bmatrix}.$$

Hence, workers are very likely to remain workers in the next year, while entrepreneurs have a much higher probability (9.4%) of moving into the group of workers in the next year. Finally, we consider the 4×4 transition matrix for workers and entrepreneurs of both productivity levels. It is

$$P = \begin{array}{c} \\ \text{w/low} \\ \text{w/high} \\ \text{e/low} \\ \text{e/high} \end{array} \begin{bmatrix} \text{worker/low} & \text{worker/high} & \text{entre/low} & \text{entre/high} \\ 0.8479 & 0.1427 & 0.0074 & 0.0020 \\ 0.1429 & 0.8520 & 0.0014 & 0.0036 \\ 0.0932 & 0.0223 & 0.6945 & 0.1901 \\ 0.0256 & 0.0488 & 0.1797 & 0.7459 \end{bmatrix}. \quad (1)$$

For example, the probability that a low-productivity worker becomes a high-productivity entrepreneur in the next period is extremely small (0.002). In each row, the highest probability is on the diagonal element, implying that remaining in the current state is the most likely outcome in the next period. Workers are even more persistent in this regard than entrepreneurs.

3. A simple benchmark model

In this section, we introduce a simple benchmark model without mobility. The purpose is twofold. First, it allows us to calibrate the individual productivity-age profiles for all types of workers and entrepreneurs. Second, we conduct a policy experiment of a pension reform that broadens the contribution base to include entrepreneurs in Section 5. The role of mobility between entrepreneurs and workers and between efficiency groups is considered separately in Section 6.

3.1. Demographics

Households live a maximum of 80 periods. Periods are equal to one year. Households are born at age 1 (corresponding to a real-life age of 20). All agents of age s survive until age $s+1$ with probability ϕ_s, with $\phi_{80} = 0$. Let N_t denote the number of households at period t. We assume that the population grows at a constant rate, $\frac{N_{t+1}}{N_t} = 1 + n$.

The first 45 periods, agents are working, while in the last 35 periods, they are retired and receive pensions if they have paid social security contributions while young.

3.2. Preferences

Households maximize expected lifetime utility at age 1 in period t:

$$\sum_{s=1}^{80} \beta^{s-1} \left(\prod_{j=1}^{s-1} \phi_j \right) u(c_{t+s-1}^s, 1 - l_{t+s-1}^s) \tag{2}$$

where c_t^s and l_t^s denote consumption and labor, respectively, at age s in period t. The total time endowment is equal to one and allocated between leisure $1-l$ and work l.

Instantaneous utility is assumed to be Cobb-Douglas in consumption c and leisure $1-l$:

$$u(c,l) = \frac{\left(c^{\gamma}(1-l)^{1-\gamma}\right)^{1-\sigma} - 1}{1-\sigma}, \tag{3}$$

where $1/\sigma$ denotes the intertemporal elasticity of substitution and γ is the weight of consumption in utility.

We consider two different kinds of households, workers (indexed by "w") and entrepreneurs (indexed by "e"). Households are born as either workers or entrepreneurs and remain the same type throughout their life. The share of entrepreneurs, ψ, among the newborn generation at age $s=1$ (and in the whole population) is set so that it is equal to that of the German population. Entrepreneurs inelastically supply working time $\bar{l} = 0.3$, while the workers' labor supply is endogenous.

Following Cagetti and de Nardi (2009), we assume that each person is characterized by two different types of abilities: entrepreneurial productivity at age s, $\theta^{e,s,j}$, and worker productivity at age s, $\theta^{w,s,j}$, $j=1,2$. In our simplified model, we assume that the productivity type j is constant.

Worker. The s-year-old worker receives total gross labor income, $\theta^{w,s,j} l_t^{s,j} w_t$, in period t, which is equal to the product of his productivity $\theta^{w,s,j}$, his working time $l_t^{s,j}$, and the wage per efficiency unit w_t. He also earns interest r_t on his wealth $a_t^{w,s,j}$. In addition, all households receive transfers tr_t from the government. In old age, the worker receives pensions $p_t^{s,j}$ that depend on his productivity type $j \in \{1,2\}$. For $s \leq 45$, pensions are zero, $p_t^{s,j} \equiv 0$. In our simplified model, we assume that pensions are exogenous to the worker. Consumption is taxed at rate τ^c.

In addition, we assume that the worker pays wage income taxes τ^l and social security contributions τ^p on his wage income as well as taxes τ^r on his capital income such that the budget constraint is presented by

$$a_{t+1}^{w,s+1,j} = (1-\tau^l-\tau^p)\theta^{w,s,j} l_t^{s,j} w_t + [1+(1-\tau^r)r_t]\, a_t^{w,s,j} \\ + tr_t + p_t^{s,j} - (1+\tau^c)c_t^{w,s,j}. \tag{4}$$

Entrepreneur. At the beginning of each period t, the entrepreneur at age s with productivity $j \in \{1,2\}$ decides how much she invests in working capital $k_t^{s,j}$. Her production net of depreciation is presented by

$$f(k_t^{s,j}, \bar{l}) = \theta^{e,s,j}(k_t^{s,j})^{\alpha}\left(A_t\bar{l}\right)^{1-\alpha} + (1-\delta)k_t^{s,j}, \tag{5}$$

where $\bar{l} \equiv 0.3$ denotes her constant labor supply and aggregate labor productivity A_t grows at the exogenous growth rate g:

$$A_t = (1+g)A_{t-1}.$$

Her borrowing costs amount to $r_t k_t^{s,j}$ so that her first-order condition with respect to the capital stock $k_t^{s,j}$ is presented by

$$r_t = \alpha\theta^{e,s,j}(k_t^{s,j})^{\alpha-1}\left(A_t\bar{l}\right)^{1-\alpha} - \delta.$$

The taxable entrepreneurial income amounts to

$$y_t^{e,s,j} = \theta^{e,s,j}(k_t^{s,j})^{\alpha}(A_t\bar{l})^{1-\alpha} - r_t k_t^{s,j} - \delta k_t^{s,j}. \tag{6}$$

In our benchmark model, we again assume that entrepreneurs pay a constant labor income tax on their non-interest income. For simplification, we assume that the tax rate is equal to the labor income tax rate τ^l so that the budget constraint of the s-year-old entrepreneur with productivity j and wealth $a_t^{e,s,j}$ in period t is given by

$$a_{t+1}^{e,s+1,j} = [1+(1-\tau^r)r_t]\, a_t^{e,s,j} + (1-\tau^l)y_t^{e,s,j} + tr_t - (1+\tau^c)c_t^{e,s,j}.$$

Note that entrepreneurs do not contribute to the social security system in Germany and do not receive a pension from the public pay-as-you-go pension system. In our calibration, we identify $y_t^{e,s,j}$ with the earnings reported by entrepreneurs in the SOEP.

3.3. Credit market

Entrepreneurs borrow or lend an amount $(k_t^{s,j} - a_t^{e,s,j})$ from a financial intermediary at rate r_t, workers lend an amount $a_t^{w,s,j}$, and the corporate sector and the government borrow amounts $K_{c,t}$ and D_t, respectively. The equilibrium in the credit market where the credit demand is equal to the credit supply is described below.

3.4. Technology

In the non-entrepreneurial (corporate) sector, output is produced with the help of capital and effective labor, according to the standard Cobb-Douglas function:

$$Y_{c,t} = A_t K_{c,t}^{\alpha} L_{c,t}^{1-\alpha}$$

where $K_{c,t}$ and $L_{c,t}$ denote total capital and labor input in the non-entrepreneurial sector, respectively. Capital $K_{c,t}$ also depreciates at rate δ.

Firms are competitive and maximize profits $\Pi_t = Y_{c,t} - r_t K_{c,t} - w_t L_{c,t} - \delta K_{c,t}$ such that factor prices are given by

$$w_t = (1-\alpha) A_t K_{c,t}^{\alpha} L_{c,t}^{-\alpha}, \tag{7a}$$

$$r_t = \alpha A_t K_{c,t}^{\alpha-1} L_{c,t}^{1-\alpha} - \delta. \tag{7b}$$

In equilibrium, corporate profits are equal to zero.

3.5. Government and social security

In our simplified model, the government levies income taxes τ^l and τ^r on labor and capital income, respectively. In addition, the government confiscates all accidental bequests Beq_t. It pays transfers Tr_t, provides a certain level G_t

of total public expenditures, and pays interest to the accumulated debt D_t. In each period, the government budget is financed by issuing government debt:

$$Tr_t + G_t + r_t D_t - Tax_t - Beq_t = D_{t+1} - D_t. \tag{8}$$

The social security authority collects contributions at rate τ^p from workers, which it uses to finance pensions. We assume in accordance with the German pension system that pensions are perfectly related to contributions such that $p_t^{s,j} = p_t^j$ for all $s > 45$ and, for $j \in \{1, 2\}$,

$$\sum_{s=46}^{80} p_t^j \, \mu(s, j, w, t) = \sum_{s=1}^{45} \tau^p \theta^{w,s,j} l_t^{s,j} w_t \, \mu(s, j, w, t),$$

where $\mu(s, j, w, t)$ denotes the measure of the s-year-old worker with productivity type j in period t. Similarly, $\mu(s, j, e, t)$ denotes the measure of the s-year-old entrepreneur with productivity type j in period t.

3.6. Equilibrium conditions

The equilibrium is described by the following conditions:

1. Aggregate consistency conditions in the labor market imply that the aggregate labor supply in the corporate and entrepreneurial sector, $L_{c,t}$ and $L_{e,t}$, is equal to the sum of the workers' and entrepreneurs' individual (effective) labor supplies:

$$L_{c,t} = \sum_{s=1}^{45} \sum_{j=1,2} \theta^{w,s,j} \, l_t^{s,j} \, \mu(s, j, w, t), \tag{9a}$$

$$L_{e,t} = \sum_{s=1}^{45} \sum_{j=1,2} \theta^{e,s,j} \, \bar{l} \, \mu(s, j, w, t). \tag{9b}$$

2. The capital stock in the entrepreneurial sector is equal to

$$K_{e,t} = \sum_{s=1}^{45} \sum_{j=1,2} k_t^{s,j} \, \mu(s, j, e, t).$$

3. Aggregate gross production in the entrepreneurial sector amounts to

$$Y_{e,t} = \sum_{s=1}^{45} \sum_{j=1,2} \theta^{e,s,j} (k_t^{s,j})^{\alpha} (A_t \bar{l})^{1-\alpha} \mu(s, j, e, t).$$

4. Equilibrium in the capital market implies that aggregate wealth Ω_t is allocated to aggregate capital and government debt:

$$\Omega_t = K_{e,t} + K_{c,t} + D_t, \tag{10}$$

with

$$\begin{aligned}\Omega_t &= \sum_{s=1}^{80} \sum_{j=1,2} a_t^{w,s,j} \mu(s,j,w,t) + \sum_{s=1}^{80} \sum_{j=1,2} a_t^{e,s,j} \mu(s,j,e,t) \\ &= \Omega_t^w + \Omega_t^e.\end{aligned}$$

The aggregate capital stock is given by

$$K_t = K_{e,t} + K_{c,t}.$$

5. In the goods market equilibrium, production is equal to (public and private) consumption demand and investment:

$$Y_{c,t} + Y_{e,t} = C_t + G_t + K_{t+1} - (1-\delta)K_t.$$

6. Tax revenues are composed of interest income, labor income and consumption taxes according to

$$Tax_t = \tau^r r_t \Omega_t + \tau^l L_{c,t} w_t + \tau^l \sum_{s=1}^{45} \sum_{j=1,2} y_t^{e,s,j} \mu(s,j,e,t) + \tau^c C_t.$$

7. Aggregate pensions P_t amount to

$$P_t = \sum_{s=46}^{80} \sum_{j=1,2} p_t^j \; \mu(s,j,w,t)$$

and the budget of the social security authority is balanced

$$P_t = \tau^p w_t L_{c,t}.$$

8. Accidental bequests amount to

$$\begin{aligned}Beq_{t+1} = &\sum_{s=1}^{80} \sum_{j=1,2} (1-\phi_s) \left[(1+(1-\tau^r)r_{t+1}) a_{t+1}^{w,s+1,j}\right] \mu(s,j,w,t) \\ &+ \sum_{s=1}^{80} \sum_{j=1,2} (1-\phi_s) \left[(1+(1-\tau^r)r_{t+1}) a_{t+1}^{e,s+1,j}\right] \; \mu(s,j,e,t).\end{aligned}$$

To derive stationary values, we divide all aggregate variables X_t (with the exception of aggregate labor $L_{c,t}$ and $L_{e,t}$) by the product of population N_t and aggregate labor productivity A_t, $\tilde{X}_t \equiv X_t/(N_t A_t)$. All individual variables x_t (with the exception of the individual labor supply) are transformed into stationary variables by the division of A_t, $\tilde{x}_t \equiv x_t/A_t$ For example, accidental bequests in stationary equilibrium amount to

$$(1+g)\tilde{B}eq_{t+1} = \sum_{s=1}^{80}\sum_{j=1,2}(1-\phi_s)\left[(1+(1-\tau^r)r_{t+1})\tilde{a}_{t+1}^{w,s+1,j}\right]\frac{\tilde{\mu}(s,j,w,t)}{1+n} + \sum_{s=1}^{80}\sum_{j=1,2}(1-\phi_s)\left[(1+(1-\tau^r)r_{t+1})\tilde{a}_{t+1}^{e,s+1,j}\right]\frac{\tilde{\mu}(s,j,e,t)}{1+n},$$

with the normalization of the stationary distribution $\tilde{\mu}(s,j,\epsilon,t) \equiv \mu(s,j,\epsilon,t)/N_t$:

$$\sum_{s=1}^{80}\sum_{j=1,2}\tilde{\mu}(s,j,w,t) + \sum_{s=1}^{80}\sum_{j=1,2}\tilde{\mu}(s,j,e,t) = 1.$$

3.7. Calibration of the benchmark model

We calibrate the model with respect to the characteristics of the German economy in 2020. For the demographic variables, we use our estimates from the German population. The population growth rate is set equal to -0.07% in 2020, as projected by United Nations (2015). The share of the entrepreneurs in the labor force is calibrated with the help of the SOEP data at 8% (see Section 2.1).

We choose our preference parameters as follows. The discount factor $\beta = 1.011$ is taken from Hurd (1989), who, in his estimation procedure, explicitly accounts for mortality risk. Our choice of the discount factor implies a real interest rate equal to 2.7% (which seems closer to recent empirical evidence than the standard value of 4% used in the DSGE literature during the 1990s and 2000s). The intertemporal elasticity of substitution, $1/\sigma$, is set to 1/2. The parameter $\gamma = 0.255$ is calibrated (see below) so that the average working time of the households is equal to that of the entrepreneurs (30% of available time).

The calibration of the production and fiscal parameters follows Trabandt and Uhlig (2011). In particular, we set the production elasticity of capital α, the

Parameter	Value	Description
n	-0.070%	population growth rate
β	1.011	subjective discount factor
γ	0.255	weight of utility from leisure
$1/\sigma$	1/2	intertemporal elasticity of substitution
ψ	8.0%	share of entrepreneurs in labor force
$\bar{l}$	0.3	steady state labor supply of entrepreneurs
α	0.37	share of capital income
δ	6.7%	rate of capital depreciation
G/Y	21%	share of government spending in steady state production
D/Y	62.0%	public debt-GDP ratio
τ^r	23.0%	capital income tax
$\tau^l + \tau^p$	41%	labor income + social security tax
τ^c	15.0%	consumption tax
$repl$	38.7%	pension replacement rate of pensions relative to gross labor income

Table 4: Calibration of benchmark parameters

depreciation rate δ, and the annual growth rate of output g equal to $\alpha = 0.37$, $\delta = 6.7\%$, and $g = 2.0\%$. The government share G/Y amounts to 21%, while the debt-to-GDP ratio is set to 62%. The tax rates on capital income and consumption are set to $\tau^r = 23\%$ and $\tau^c = 15\%$, respectively. The sum of the workers' taxes on social security and labor income is also taken from Trabandt and Uhlig, who apply a value of 41% to Germany. Our pension contribution rate τ^p is endogenous and amounts to 17.4%, in accordance with the empirical value of 18.6% prevailing in Germany in 2020. Therefore, the implied labor income tax rate equals $41\% - 17.4\% = 23.6\%$. Finally, government transfers relative to GDP, Tr/GDP, are computed from the fiscal budget in the steady state when the debt-to-GDP ratio is constant (and equal to 62%) so that transfers amount to 4.42% of GDP. Pensions are calibrated so that the replacement rate of pensions relative to average gross labor earnings amounts to 38.7% for each worker productivity type $j \in \{1, 2\}$. Our estimate of the replacement rate is taken from the OECD (2019).

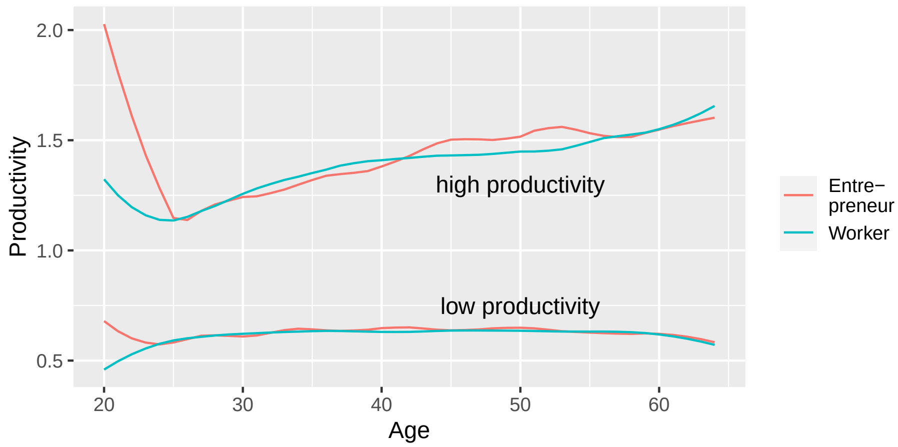

Figure 6: Age-productivity profiles

The productivity profiles of workers and entrepreneurs, $\theta^{w,s,j}$ and $\theta^{e,s,j}$, $s = 1, \ldots, 45$ for the two productivity types $j = 1, 2$, are set so that they replicate the empirical mean real hourly earnings profiles presented in Fig. 4. We normalize the age efficiency profile so that the average efficiency is equal to one. Since the mean hourly earnings of entrepreneurs is an endogenous function of their capital investment, the productivity age profile $\theta^{e,s,j}$ is not a trivial copy of the profile in Fig. 4 but needs to be calibrated endogenously with the help of the equilibrium conditions of the model.[4] Our calibrated efficiency-age profiles of workers, $\left\{\theta^{w,s,j}\right\}_{s=1}^{45}$, and entrepreneurs, $\left\{\theta^{e,s,j}\right\}_{s=1}^{45}$, are illustrated for the productivity types $j = 1, 2$ in Fig. 6. Note that the productivity levels of workers and entrepreneurs of each type almost coincide, except during the first years of working life.

4. Results

In this section, we present the age profiles of wealth a, labor supply l and consumption c of households. Due to the lack of old-age pensions, entrepreneurs

[4]The computation and calibration of the age-productivity profiles is described in more detail in Appendix A.

accumulate higher wealth and are characterized by higher income and wealth inequality, in accordance with the empirical evidence presented in Section 2.

4.1. Age profiles

Fig. 7 presents the wealth-age profiles of the workers and entrepreneurs for the low- and high-productivity types. Note that entrepreneurs accumulate much larger savings than workers because they do not receive a public pension in old age in Germany. The maximum wealth of the high-productivity entrepreneur is more than three times as high as the maximum wealth of the high-productivity worker. For workers, the precipitous fall in the survival probability during old age results in negative wealth close to the end of the lifetime.

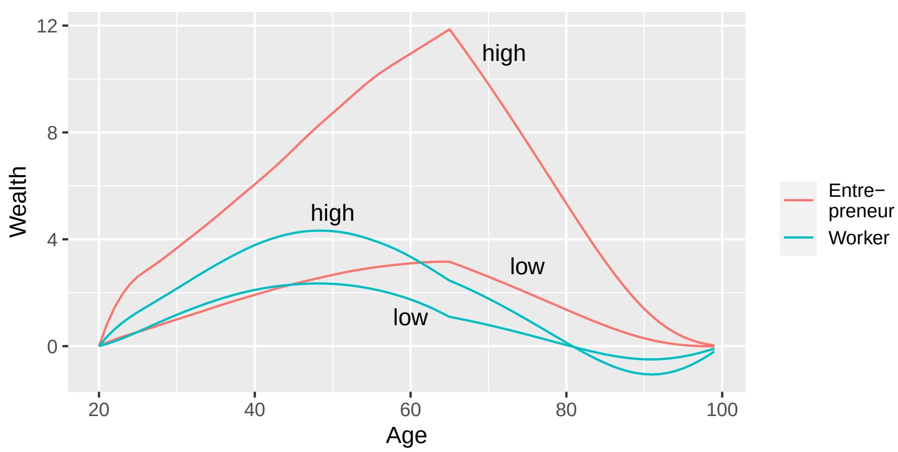

Figure 7: Wealth-age profiles

The labor supply of workers is graphed in Fig. 8 (remember that the labor supply of entrepreneurs is constant at $\bar{l} = 0.3$). The labor supply of workers declines over the lifetime as individual wealth increases. In addition, it mimics the efficiency age profile to some extent such that the labor supply is relatively high at the beginning of working life, where we observe high labor productivity. Note that low-productivity workers reduce their labor supply to zero at age 63.

The consumption-age profiles of individuals are presented in Fig. 9. In accordance with the empirical evidence provided by Krueger et al. (2010), the

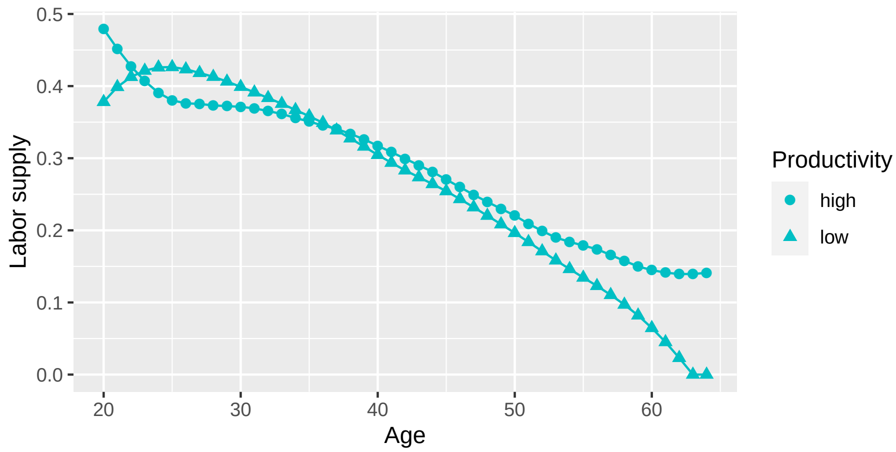

Figure 8: Labor supply-age profiles

consumption-age profile is hump-shaped. In addition, consumption jumps downwards for high-productivity workers and all entrepreneurs at the age of retirement because leisure increases to one. In an effort to smooth utility intertemporally, the household reduces consumption at this age. There is no jump in consumption for the low-productivity worker because his labor supply is already zero two periods prior to retirement.[5]

The production and capital input of entrepreneurs from ages 20 to 64 mirror the efficiency-age profiles of the two types, respectively, and are illustrated in Figs. 10 and 11.

4.2. Inequality and correlations

The Gini coefficients of earnings, income and wealth for the two subgroups – workers and entrepreneurs – are presented in Table 5. They are qualitatively in accordance with the empirical values. In particular, the Gini coefficients of the earnings, gross income and net income of workers are close to each other,

[5]To avoid the downward jump in consumption at the start of retirement, which we do not observe empirically, one could introduce consumption habits. However, to some extent, the smooth empirical consumption-age profile is also caused by the fact that households retire at different ages.

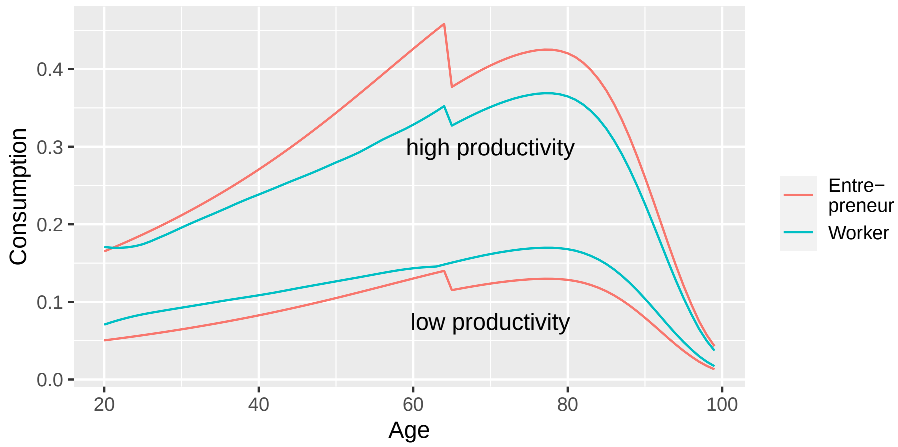

Figure 9: Consumption-age profiles

and all lie in the interval $[0.25, 0.30]$, as observed empirically. For entrepreneurs, we observe both in the model and empirically that the inequality of earnings and gross income (pre-government income) is larger than that of the workers. However, for entrepreneurs, the quantitative magnitudes of the Gini coefficients are slightly too small for earnings and gross income, while they are slightly too high for net income.[6]

With respect to the correlation of (gross) earnings and (net) wealth, our model underpredicts the correlation for workers (0.04 versus an average value of approximately 0.30 during 2002-2017) and overpredicts the correlation for entrepreneurs (0.714 versus an average value of 0.29 in the data).[7]

In our model, it is straightforward to compute lifetime utility (2) with the

[6] In our sensitivity analysis of the model in Section 6, where we introduce progressive income taxation, the Gini coefficient of net income among entrepreneurs falls to 0.30.

[7] A possible explanation for the lower empirical correlation of earnings and wealth among entrepreneurs might be the presence of credit constraints for the financing of entrepreneurs' capital investment. A possible explanation for the empirically low but statistically significant positive correlation of earnings and net wealth among workers might be the endogeneity of individual productivity. If individual productivity levels, for example, depend on the individual's investment in human capital, a positive correlation naturally arises. Similarly, if human capital is not perfectly mobile between generations but the individual's innate abilities depend on those of his or her parents, wealth-rich parents might transfer both physical and human capital to their children.

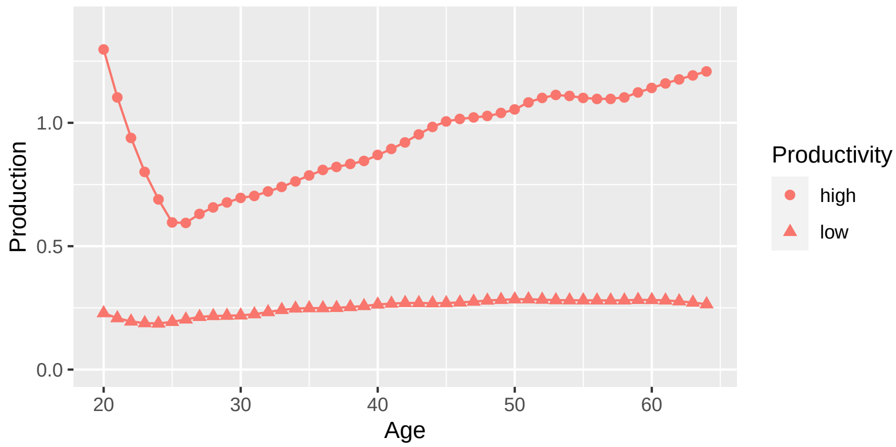

Figure 10: Production-age profiles of entrepreneurs

	Workers	Entrepreneurs
Gini coefficient		
Earnings	0.299	0.334
Gross income	0.270	0.346
Net income	0.255	0.326
Wealth	0.298	0.430
Correlation between earnings and wealth		
	0.043	0.714

Table 5: Gini coefficients and correlations in the model.

help of the consumption-age and labor supply-age profiles. As presented in Table 6, a low-productivity worker attains higher expected lifetime utility than a low-productivity entrepreneur, while the order is reversed in the case of the high-productivity type.

Expressed as the consumption equivalent change (with the low-productivity worker as the benchmark), we find that the low-productivity worker would benefit from switching positions with the high-productivity worker at the time of birth. In this case, his lifetime utility would increase by a consumption equivalent of 109%. For a low-productivity worker, the increase in consumption would even amount to 137% if he became a high-productivity entrepreneur at the time

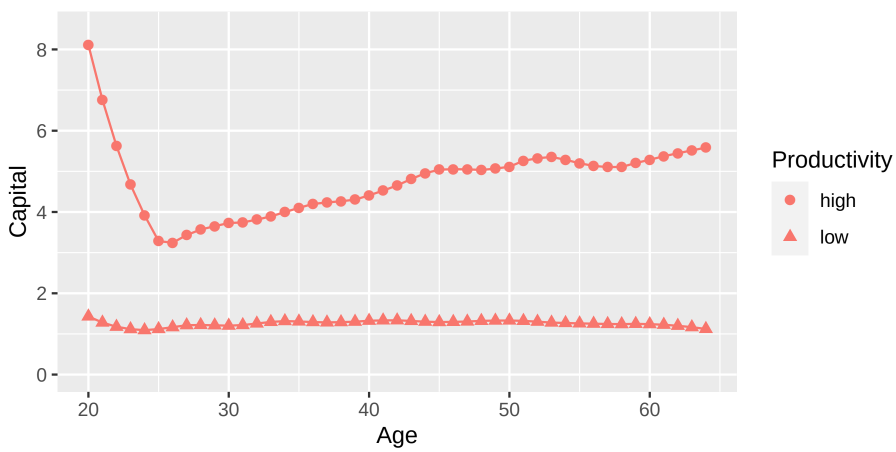

Figure 11: Capital-age profiles of entrepreneurs

	Workers	Entrepreneurs
Low-productivity type	−175.52	−190.59
	(0%)	(−27.6%)
High-productivity type	−145.31	−140.81
	(109.7%)	(137.3%)

Table 6: Lifetime utility. Consumption equivalent change in brackets.

of birth.

5. Policy experiment: PAYG pensions for entrepreneurs

In this section, we conduct a policy experiment in which entrepreneurs pay contributions to the social security system. We make the following assumptions:

1. The pension levels remain constant for workers. The contribution rate τ^p adjusts to keep the budget of the pay-as-you-go system balanced if, due to general equilibrium effects, the aggregate wage income of the workers and, hence, the tax basis changes.
2. Entrepreneurs have to pay the same social security tax τ^p on their non-interest income as the workers. The pensions of the two productivity types, $j = 1, 2$, of entrepreneurs are individually self-financed, meaning that the

contributions of the low- (high-) productivity entrepreneurs finance the pensions of all low- (high-) productivity entrepreneurs during retirement.[8]

3. We assume that the levels of government consumption, G, and public debt, D, (relative to aggregate productivity and population) remain constant under the different scenarios that we consider.
4. As a consequence of the pension reform, we observe various general equilibrium effects. For example, the wealth of entrepreneurs drops dramatically because they have to save less for old age. Therefore, tax revenues also fall. We analyze two scenarios of how the government balances the budget: i) with lower transfers tr, keeping the other tax rates τ^l, τ^r, and τ^c constant and ii) with higher labor income taxes τ^l, keeping tr, τ^r, and τ^c constant.

Our results are summarized in Table 7. In the first entry column, we present the benchmark case in which entrepreneurs do not contribute to the public pension system. In this case, the equilibrium pension contribution rate τ^p that balances the budget of the social security authority is equal to 17.46%. GDP, which is equal to the sum of the production in the corporate and entrepreneurial sectors, amounts to 0.441.

In the second column, the results are presented for the case in which entrepreneurs contribute to the social security system and the shortfall in taxes is financed by a reduction of transfers tr. Since entrepreneurs save less for old age, the capital stock in the corporate sector K^c falls as well, and therefore, wages decline. Labor supply nevertheless increases by a small amount since the income effect from lower transfers outweighs the substitution effect from lower wages. The pension contribution rate τ^p has to increase from 17.46% to 17.55% because the tax base of the pension system (the total wage income of

[8]This reflects the schedule of the German pay-as-you-go pension system, which is basically proportional to contributions and characterized by a low degree of progressivity. According to the OECD (2007), the German pension system is characterized by a progressivity index of 26.7, while the more redistributive public pension systems in the UK and Canada display progressivity indices equal to 81.2 and 86.6.

	No PAYG	PAYG entrepreneurs	
		tr adjusts	τ^l adjusts
K^c	1.567	1.500	1.398
L^c	0.178	0.181	0.175
GDP	0.441	0.438	0.419
τ^l	23.54%	23.54%	26.54%
τ^p	17.46%	17.55%	18.38%
tr	0.0197	0.0172	0.0197
Lifetime utilities			
Worker, $j=1$	−175.52	−177.34	−178.39
	(0%)	(−3.96%)	(−6.16%)
Worker, $j=2$	−145.31	−146.54	−148.12
	(0%)	(−3.25%)	(−7.24%)
Entrepreneur, $j=1$	−190.59	−174.50	−175.30
	(0%)	(+41.3%)	(+38.8%)
Entrepreneur, $j=2$	−140.81	−127.06	−128.13
	(0%)	(+49.62%)	(+44.78%)
Expected (newborn)	−160.84	−161.05	−162.33
	(0%)	(−0.51%)	(−3.55%)

Table 7: Pension reform: Entrepreneurs contribute at the same rate τ^p as workers. Consumption equivalent changes in brackets.

the workers) declines. The decrease in capital stock is stronger than the increase in effective labor, so GDP declines by 0.7%. This decline seems rather small, but remember that entrepreneurs constitute only 8% of the labor force. The welfare effects are noteworthy. While workers experience a decline in their lifetime utility, the welfare of entrepreneurs increases. Workers mainly suffer from the general equilibrium effects of lower aggregate savings.[9] The benefits for entrepreneurs from inclusion in the pay-as-you-go pension system are substantial and amount to 41% and 50% for the low- and high-productivity types, respectively.

[9]Our result, therefore, is in accordance with the results from the literature on the optimal amount of pensions cited in the Introduction, which finds an optimal low pension level. The mechanism is the same in this literature and in our model. The decline in aggregate savings and the resulting drop in wages outweighs the benefits of higher pension income in old age. In the next section, we also study the sensitivity of our results with respect to income uncertainty, which constitutes one of the main reasons for the provision of public pensions.

The right-most column considers the case in which the fiscal budget is adjusted with the help of the labor income tax rate τ^l. Notice that in this case, the decrease in savings is even larger, so the aggregate capital stock in the corporate sector falls by 10.8%, from 1.567 to 1.398. As a consequence, the fall in the wage rate is so strong that the (effective) labor supply of workers also decreases by 1.7%, from 0.178 to 0.175. The shortfall in taxes has to be financed by a substantial increase in the labor income tax rate by 3 percentage points, from 23.54% to 26.54%. Moreover, the strong decline in savings and, hence, labor income among workers also results in a large increase in the pension contribution rate from 17.46% to 18.38%. The combined distortionary effects of higher labor income taxes τ^l and pension contributions τ^p imply a significant decrease in GDP by 5.0%. In this case, again, workers suffer from the policy reform, while entrepreneurs benefit, but to a smaller extent than in the case of constant labor income taxes.

In summary, if the change in tax revenue in Germany that results from the introduction of social security for entrepreneurs cannot be financed by lower transfers – many of the transfers are guaranteed by legislation and cannot be changed easily – the resulting general equilibrium effects are extremely detrimental to production and efficiency due to the distortionary effects of the higher tax wedge. Even in the case of lower transfers, the expected lifetime utility of the newborns who do not yet know their type j or whether they are workers or entrepreneurs declines, from -160.84 to -161.05 (last line in Table 7), or approximately 0.5% of total consumption.

6. Extensions

In this section, we extend our benchmark model to provide a more realistic description of the income and wealth distribution and dynamics in the German economy (both before and after taxes). Therefore, we include the following additional elements in our model:[10]

[10]The model is described in more detail in the Appendix.

1. Income mobility:

   We use the Markov transition matrix (1) to model the dynamics of working-age households both between productivity classes and employment statuses (worker/entrepreneur).

2. Progressive income taxation:

   Following Holter et al. (2019), after-tax labor income y is given by

$$y = \zeta_0 \left(\frac{\hat{y}}{\bar{y}} \right)^{1-\zeta_1}, \tag{12}$$

   where $\hat{y}$ ($\bar{y}$) denotes the (average) before-tax labor income. In addition, we impose a maximum tax rate of 42%, which is the upper threshold in the German income tax schedule. All household types (workers, entrepreneurs, and retirees) have to pay income taxes on non-interest income.

3. Contributions-related pension entitlements:

   As a close approximation to the German pay-as-you-go pension system, pensions are proportional to the average contributions during working life. Entrepreneurs do not contribute to the pension system in the benchmark case.

4. Credit constraint:

   Households cannot accumulate debt during their lifetime.

This section is structured in three parts. First, we compare the allocation in the extended model to that in the benchmark model in Section 4. Second, we consider the effects of two pension reform proposals. In the first scenario, entrepreneurs also contribute to the pension system, and pensions are proportional to contributions. In the second scenario, we abandon the strict proportionality in the pension system with respect to the contributions and consider lump-sum pensions instead. Third, we analyze the long-run effects of aging on the sustainability of pensions and apply the projected population parameters for the year 2050 to our model.

6.1. Allocation in the extended model with mobility

Table 8 presents the results for the extended model. As a consequence of the new elements in the model, savings, aggregate labor supply and GDP all decrease in comparison with the benchmark model presented in Table 7. There are mainly two new opposing effects on the aggregate labor supply in the extended model. First, pensions are based on contributions. Therefore, the individual labor supply increases for all workers. Second, income is taxed progressively. As a consequence, the labor supply of high-productivity workers decreases, while that of low-productivity workers increases.[11] In summary, aggregate effective labor L^c in the corporate sector declines from 0.178 to 0.168. As a consequence, aggregate labor income and, hence, savings also decline. Due to income uncertainty, however, households increase their precautionary savings. The overall effect on savings is also negative such that the capital in the corporate sector, K^c, declines from 1.567 to 1.316. Since both production factors in the corporate sector decline, we also notice a drop in GDP by 8.8%, from 0.441 to 0.402.

With respect to the fiscal parameters, we observe that transfers increase from 0.0197 to 0.0651. This increase results from the higher tax revenue due to progressive taxation.[12] The increase in transfers also reinforces the drop in savings and aggregate labor supply. In addition, we observe a rise in the social security contribution rate from 17.46% to 22.04%, which stems from our calibration procedure in the benchmark and extended models. In both cases, we assume the same replacement rate of pensions with respect to the average labor income of individuals who work 30% of their available time. Since workers supply less labor in the case of progressive income taxes and the contribution base of pensions declines, τ^p has to increase.

The measures of inequality and correlation are also affected by the introduction of these new, more realistic elements to the model. Comparing Table 9

[11] In addition, as we will argue below, transfers increase due to higher tax revenue. Because of this income effect, the labor supply of low-productivity workers also decreases in equilibrium.

[12] The tax function is calibrated so that the average income pays the same income tax rate of 23.54% in both models.

	No PAYG	PAYG entrepreneurs	
		contributions-based pension	lump-sum pensions
K^c	1.316	1.223	1.330
L^c	0.168	0.167	0.157
GDP	0.402	0.390	0.389
τ^p	20.17%	22.04%	20.28%
tr	0.0697	0.0651	0.0652
Lifetime utilities			
Worker, $j = 1$	−157.96	−159.59	−154.38
	(0%)	(−3.95%)	(+9.41%)
Worker, $j = 2$	−155.29	−156.61	−155.63
	(0%)	(−3.26%)	(−0.86%)
Entrepreneur, $j = 1$	−156.48	−159.74	−155.90
	(0%)	(−7.77%)	(+1.47%)
Entrepreneur, $j = 2$	−151.42	−155.33	−152.18
	(0%)	(−9.51%)	(−1.94%)
Expected (newborn)	−156.32	−158.33	−154.38
	(0%)	(−4.89%)	(+5.02%)

Table 8: Sensitivity study: Model with mobility, tax progression, and contributions-based pensions. Consumption-equivalent changes in brackets.

with Table 5, we notice that the Gini coefficient of earnings among workers increases dramatically, from 0.299 to 0.491, and is now much more similar to the empirical evidence presented in Fig. 5. The main reason for this increase stems from the uncertainty of income and the mobility of workers. For example, a worker who has been highly productive over a longer period and has accumulated large savings reduces his labor supply to almost zero when he becomes low productivity (with a probability of 14% each year). Conversely, a worker who has been characterized by low productivity for a long time in his working life increases his labor supply once he becomes highly productive. Accordingly, the intertemporal substitution of labor increases significantly. Note that a worker in Germany changes his productivity type every seven years on average over his lifetime.

In Table 9, we also provide the Gini coefficients of gross and net income for both groups: workers and entrepreneurs. Gross income is defined as all

	Workers	Entrepreneurs
Gini coefficients		
Earnings	0.491	0.336
Wealth	0.355	0.357
Gross income	0.407	0.308
Net income	0.389	0.302
Correlation between earnings and wealth		
	-0.077	0.136

Table 9: Gini coefficients and correlations in the model with mobility and progressive taxation.

income, including interest income but excluding transfers; net income is defined as gross income plus transfers minus labor and capital income taxes. For this reason, gross income is less concentrated than earnings for both workers and entrepreneurs because higher wealth reduces the labor supply and, hence, labor income ceteris paribus. Due to the progressive tax schedule, net income is less concentrated than gross income, and the Gini coefficients fall from 0.407 (0.308) to 0.389 (0.302) for workers (entrepreneurs).[13] Our values differ slightly from the empirical values for the German economy presented in Fig. 5. The inequality of income among the workers is slightly higher in our model than in the empirical results (e.g., the Gini coefficients of net income amount to 0.308 in the model and 0.28 in Germany), while the model underpredicts the concentration of income among entrepreneurs (e.g., the Gini coefficients of net income amount to 0.302 and 0.35, respectively). The concentration of wealth as measured by the Gini coefficients amounts to 0.355 and 0.357 for workers and entrepreneurs, respectively. With respect to the correlation of earnings and wealth, we also find a small value. While empirically, the correlations amount to approximately 0.30 for workers and entrepreneurs during 2002-2017, we obtain somewhat lower correlations of -0.08 and 0.14, respectively.

[13] For the aggregate economy including retirees, we find Gini coefficients of wealth, earnings, gross and net income equal to 0.409, 0.494, 0.463 and 0.447, respectively.

6.2. Pension reform proposals

In the second and third entry columns of Table 8, we present the results for the case of a pension reform in which entrepreneurs also contribute to the pay-as-you-go system.[14] In the second entry column, all pensions in the economy are paid proportional to the individual's contributions to the pension system. As in the case without mobility, aggregate savings drop substantially so that the capital stock in the corporate sector, K^c, decreases by 7.1%, from 1.316 to 1.223. Since the aggregate labor supply is almost unaffected, output in the corporate sector, Y_c, falls by 2.6%, whereas GDP falls by a larger share (3.0%) due to the decrease in the capital stock among entrepreneurs. We also notice that the contribution rate τ^p has to increase from 20.17% to 22.04% to finance the pension system.

In the case of lump-sum pensions (third entry column), aggregate savings increase substantially compared to the case of contributions-based pensions (for both workers and entrepreneurs) presented in the second entry column. Since income-rich workers and entrepreneurs with high productivity receive a smaller lump-sum pension, they increase their savings for retirement. Consequently, aggregate capital in the corporate sector, K^c, increases from 1.223 to 1.330. However, the aggregate labor supply of workers, L^c, declines from 0.167 to 0.157 since pensions no longer depend on contributions. As the drop in labor is more pronounced than the increase in savings, GDP is even lower in the case of lump-sum pensions than in the case of contributions-based pensions and falls by 0.3%, from 0.390 to 0.389.

The welfare effects of the two pension reform proposals are straightforward. In the case of contributions-based pensions, where entrepreneurs contribute to the PAYG system, welfare as measured by expected lifetime utility declines significantly for all types of agents. If you compare the first and second entry columns, the lifetime utilities of all household types, including workers and

[14] Again, we assume that the levels of government debt and consumption are constant in comparison to the case where entrepreneurs do not contribute to the PAYG system.

entrepreneurs, decline. For newborn agents who do not know their type yet, expected lifetime utility falls from -156.32 to -158.33, corresponding to a decrease in total consumption of 4.9%. As an obvious explanation, the general equilibrium effects of lower capital and aggregate labor supply reduce aggregate income and, hence, utility.

When we investigate the welfare effects of a pension reform that introduces lump-sum benefits irrespective of the contributions, we find that welfare increases in comparison with the case of contributions-based pensions (compare the second and the third entry columns). The expected lifetime utility of a newborn increases from -158.33 to -154.38, corresponding to an increase in total consumption equal to 9.9%. Our results confirm the findings of Fehr et al. (2013) that the optimal pension schedule should be more progressive and include a large lump-sum component.

6.3. Entrepreneurship and aging

In the following, we consider the effects of aging on equilibrium values of capital, labor and income and the sustainability of public PAYG pensions. We focus on the demographics projected for the German economy in the year 2050. Therefore, we apply the population growth rate and survival probabilities projected by United Nations (2015) but keep the other parameters of the extended model unchanged. In particular, we keep the tax and pension parameters at their respective values in 2020.

Heer et al. (2020) show that the German pension system cannot be financed by higher social security taxes on labor income beyond 2035 if 1) the effective retirement age remains at 65[15] and 2) the pension replacement rate with respect to wage income remains at 38.7%, while the social security tax rate has to be adjusted to finance public pensions. Even if the social security tax is raised to the value associated with the top of the Laffer curve, there is a shortfall in

[15] According to the OECD (2018), the average effective retirement age in Germany amounted to 64.0 years during 2013-2018.

revenues at the social security authority.[16] As a sensitivity analysis for their study, we investigate whether social security can be financed with the help of additional contributors in the form of entrepreneurs. However, we confirm the results of Heer et al. (2020) that the maximum increase in revenues from taxes on labor income is insufficient to pay for the increase in public pension expenditures caused by the higher old-age dependency ratio in 2050.[17]

	2020	2050	
Entrepreneurs:	No PAYG	No PAYG	PAYG
Retirement age:	65	70	70
K^c	1.316	1.358	1.309
L^c	0.168	0.166	0.166
GDP	0.402	0.405	0.399
τ^p	20.17%	22.25%	22.27%
tr	0.0697	0.0686	0.0666
Lifetime utilities			
Worker, $j=1$	−157.96	−174.10	−174.80
		(0%)	(−1.56%)
Worker, $j=2$	−155.29	−171.44	−171.90
		(0%)	(−1.04%)
Entrepreneur, $j=1$	−156.48	−172.15	−174.99
		(0%)	(−6.22%)
Entrepreneur, $j=2$	−151.42	−166.79	−170.67
		(0%)	(−8.62%)
Expected (newborn)	−156.32	−172.39	−173.29
		(0%)	(−2.02%)

Table 10: Aging and the sustainability of pensions.

Therefore, we consider the effects of aging under the assumption that the effective retirement age can be increased to 70 years in Germany by 2050. The results for the case where the entrepreneurs (do not) contribute to the social security system in 2050 are summarized in the third (second) entry column

[16] Even in 2018, the federal budget already contributed approximately 68 billion Euros to the financing of social security expenditures, which amounted to 2.04% of GDP.

[17] In Germany, the old-age dependency ratio, defined as the ratio of those aged 65+ to those aged 20-64, is projected to increase from 38% in 2020 to 64% in 2050 (see United Nations, 2015).

of Table 10. For comparison, we also replicate the benchmark case in 2020, keeping in mind that in this case, the retirement age is lower and amounts only to 65. Strikingly, the increase in retirement to the age 70 almost balances the negative effect of aging, and the aggregate labor supply declines only from 0.168 to 0.166. Therefore, the pension contribution rate only has to increase from 20.17% in 2020 to 22.25% in 2050. The workers who receive an income over a longer period of 50 instead of 45 years increase their savings for old age for two reasons. First, their income over their whole working life increases. Second, the survival probabilities are higher in old age so that the weight on instantaneous utility (the survival probabilities times the discount factor) increases. These two savings-increasing factors outweigh the negative effect that stems from a shorter maximum retirement life, which falls from 35 to 30 years. We therefore conclude – in accordance with Heer et al. (2020) – that the German pension system will be sustainable in 2050 if the retirement age is increased to 70.

In the second column, we present the case in which entrepreneurs contribute to the social security system as well. In this case, the main effect on the aggregate variables is again the reduction in aggregate savings and, hence, the capital stock in the corporate sector, K^c which falls from 1.358 to 1.309. The aggregate labor supply in the corporate sector L^c is basically unaffected so that GDP falls by only 1.5%. With respect to welfare, our results in the previous section are shown to be robust to demographic change. The integration of entrepreneurs into the German pension system reduces the welfare of all productivity types and the average newborn. Expected lifetime utility falls from -172.39 to -172.39, corresponding to a consumption equivalent of 2.0%.[18]

[18] We refrain from comparing the expected lifetime utility in the years 2020 and 2050. In our model with a calibrated value of the risk aversion coefficient $\sigma > 1$, an increase in longevity results in a decline in expected lifetime utility, even if the consumption-age and labor-supply age profiles remain constant.

7. Conclusion

The public pension system in Germany is not sustainable as it is. Increasing the contribution rate on gross wages to the rate associated with maximum revenue will not restore balance in the pension system in the coming decades. The questions of whether broadening the contribution base to include entrepreneurs will help to establish fiscal sustainability – and what are the economic consequences and welfare effects of such a policy – arise.

We present a model that replicates German age-specific entrepreneurship and heterogeneity among both entrepreneurs and workers and between cohorts. We show that the inclusion of entrepreneurs neither helps to establish fiscal sustainability of the pension system nor increases welfare. Only if the retirement age is raised to 70 by the year 2050 will contributions be sufficient to finance pensions. The contribution rate that will balance the social security authority's budget will amount to 22.3% by then and is rather insensitive to the inclusion of entrepreneurs in the social security system.

As a consequence of social security contributions, entrepreneurs will have reduced net income and, hence, lower incentives to invest in their business; therefore, capital in the entrepreneurial sector will decline substantially. In general equilibrium, the accompanying decline in GDP will amount to 3.0% and average expected welfare of the newborn will be reduced by 4.9% of total consumption.

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Appendix A. Computation of the benchmark model

To compute the steady state of the benchmark, we must compute the solution to a nonlinear equations system of 412 variables consisting of the $4 \times 79 = 316$ individual assets (remember that $a^1 = 0$), the $2 \times 45 = 90$ individual labor supply of workers, and 6 aggregate variables K_c, L_c, Tr, $p^{s,1}$, $p^{s,2}$, τ^p, where $p^{s,1}$ and $p^{s,2}$ denote the (constant) pensions for workers of type $j = 1, 2$ at age $s = 46, \ldots, 80$. All other model variables can be computed with the help of these variables. For example, the factor prices can be computed with the help of K_c and L_c, while the individual consumption profiles follow from the budget constraint. The corresponding endogenous equations are the Euler equations, the workers' first-order conditions with respect to labor, the entrepreneurs' first-order conditions with respect to investment, and the aggregate consistency equations.

In the calibration procedure, we also have to add the 91 parameters γ and $\theta^{e,s,j}$. The corresponding equations are the equilibrium condition in which the average labor supply of the worker is equal to 0.3 and the calibration conditions in which the earnings profiles of the entrepreneurs are equal to their empirical values. In summary, we have to estimate a nonlinear equations system using 513 (!) variables.

This is a nontrivial task. Producing an initial value for the Newton-Rhapson algorithm is possible only with the help of a stepwise procedure. In particular, we begin with a simple 20-period model with an exogenous labor supply of workers equal to 0.3 and exogenous aggregate variables. Next, we add endogenous labor. In the following step, we include the aggregate state variables and the corresponding equilibrium conditions. We continue to iterate over the computation of this model by adding one cohort of workers in each step. To come up with an initial value of the asset and the labor supply in the new step, we use the value of the oldest cohort in the previous step. After computing the model for all workers, we continue to include one cohort of retirees in each step and endogenize the aggregate variables for the pension and social security contribution rates. Next, we include the preference parameter γ. Finally, we include the $\theta^{e,s,j}$ in the computation by introducing one cohort value s in each step. We find this procedure to be sensitive to the order in which we introduce the endogenous variables; thus, it implies a great deal of trial and error. The computational time for the whole calibration amounts to approximately 37 minutes on a 64-BIT system with 32 MB RAM and an Intel(R) Xeon(R) 2.90 GHz processor.

Appendix B. The large-scale OLG model with mobility, progressive income taxes and contributions-related pensions

The extended large-scale OLG model of Section 6 builds upon the model in Section 3. In particular, we introduce 1) progressive income taxes, 2) contributions-based pensions, 3) mobility in and out of entrepreneurship and between productivity classes and 4) a credit constraint $a \geq 0$ on wealth. However, we keep the modeling of the demographics as presented in Section 3.

Appendix B.1. Progressive income taxation

Following Holter et al. (2019), we model the progressive taxation of income from labor, while the capital income tax τ^r, the tax on social security τ^p and the consumption tax τ^c are imposed as flat-rate taxes. Let $\hat{y}$ denote the pretax labor income of the individual worker, entrepreneur or retiree. Accordingly, average income amounts to $\bar{y}$ among the income-tax payers (labor force). After-tax labor income y is given by (12) where ζ_0 and ζ_1 denote the tax level parameter and the tax progressivity parameter, respectively. Taxes on labor are given by

$$T(\hat{y}) = \hat{y} - y.$$

This functional form of the tax function has the advantage that the level ζ_0 can be changed without affecting tax progressivity ζ_1.

Appendix B.2. Contribution-based pensions

The pension system is a pay-as-you-go system. The social security authority collects contributions at rate τ^p from workers. The pension depends on accumulated contributions x during the working periods. In our model, we use the average rather than the total accumulated contributions (the so-called sum of earnings points in the German pension system) to simplify the computation, while the result, of course, is unchanged.[19] Therefore, the average accumulated earnings of the $s+1$-year-old household at the beginning of period $t+1$ are summarized by the accounting variable at age $s+1$, x_{t+1}^{s+1}, as follows:

$$x_{t+1}^{s+1} = \begin{cases} \frac{(s-1)x_t^s + \tau^p \hat{y}_t}{s} & s = 1, \ldots, 45 \\ x_t^s & s = 46, \ldots, 80, \end{cases} \tag{B.1}$$

with initial cumulated contributions equal to zero at the beginning of the life, $x_t^1 = 0$. Note that workers do not accrue interest on their social security payments in Germany.

Pensions are paid proportionally to accumulated average contributions irrespective of the household type at the end of the working period and are subject to taxes $T(\hat{y})$ on non-interest income.

Appendix B.3. Mobility

We assume that the transition between the two employment types, $\epsilon \in \{w, e\}$, and productivity types $j = 1, 2$, is governed by a Markov process. The transition probabilities are described by a matrix $\Pi(\epsilon', j'|\epsilon, j)$, where ϵ' and j' denote the next-period employment and productivity type, respectively.

Appendix B.4. Households' optimization problem

In the following, we will describe the optimization problem for the working household, $s \leq 45$ and the retired household, $s > 45$, in turn. Let $z = (a, x, j, \epsilon, s)$ denote the individual state vector.[20] $V(z)$ denotes the value function of the household.

[19] To convert the average accumulated to the total accumulated contributions, just multiply the former by the number of working periods n_w.

[20] In the following, we drop the index for the period t whenever it does not imply ambiguity.

The young's problem.. If a young household at age s with wealth a is a worker, $\epsilon = w$, with productivity type j, he maximizes his value function

$$V(a,x,j,w,s) = \max_{c,l,a'} \left\{ u(c,l) + \beta\phi_s \sum_{\epsilon',j'} \Pi(\epsilon',j'|w,j)\, V(a',x',j',\epsilon',s+1) \right\},$$

subject to the following constraints

$$a' = (1-\tau^p)\theta^{w,s,j} lw + [1+(1-\tau^r)r]\, a + tr - T(\theta^{w,s,j} lw) - (1+\tau^c)c, \tag{B.2a}$$

$$x' = \frac{(s-1)x + \tau^p \theta^{w,s,j} wl}{s}, \tag{B.2b}$$

$$a' \geq 0, \tag{B.2c}$$

$$0 \leq l \leq 1, \tag{B.2d}$$

where a' denotes the next-period assets, and instantaneous utility is presented by (3).

For the entrepreneur, the Bellman equation is presented by

$$V(a,x,j,e,s) = \max_{c,a',k} \left\{ u(c,\bar{l}) + \beta\phi_s \sum_{\epsilon',j'} \Pi(\epsilon',j'|e,j)\, V(a',x',j',\epsilon',s+1) \right\},$$

subject to:

$$a' = \hat{y} + [1+(1-\tau^r)r]\, a + tr - T(\hat{y}) - (1+\tau^c)c, \tag{B.3a}$$

$$\hat{y} = \theta^{e,s,j} k^\alpha (A\bar{l})^{1-\alpha} - rk - \delta k, \tag{B.3b}$$

$$x' = \frac{(s-1)x}{s}, \tag{B.3c}$$

$$a' \geq 0. \tag{B.3d}$$

The old's problem.. During retirement, the household neither works nor becomes an entrepreneur. His working type ϵ and productivity type j remain constant but do not have any effect on his income or utility. Instead, the household receives a pension $p(x)$ depending on his contributions x during working life. For $s \geq 46$, his value function is given by

$$V(a,x,j,\epsilon,s) = \max_{c,a'} \left\{ u(c,0) + \beta\phi_s\, V(a',x,j,\epsilon,s+1) \right\},$$

subject to

$$a' = p(x) + [1 + (1 - \tau_r)r]a + tr - T(p(x)) - c, \tag{B.4a}$$

$$a' \geq 0. \tag{B.4b}$$

Note that the state variable x (average contributions to the pension system) remains constant. In addition, the retired worker pays labor income taxes on his pensions.

Appendix B.5. Stationary equilibrium

To express the equilibrium in terms of stationary variables, we must divide aggregate variables by the mass of the total population N_t and aggregate labor productivity A_t. Therefore, we define the following stationary variables, $\tilde{X}_t \equiv \frac{X_t}{A_t N_t}$, for the aggregate variables $X \in \{P, Tr, G, D, Beq, Tax, Y, Y_c, Y_e, K, K_c, K_e, C, \Omega, \}$ and $\tilde{L}^c = L^c/N_t$ and $\tilde{L}^e = L^e/N_t$ for the aggregate labor supply in the corporate and entrepreneurial sectors. The individual variables, $\tilde{x}_t = x_t/A_t$, (except for the labor supply) are made stationary by the division of aggregate labor productivity A_t. Let $\tilde{\mu}$ denote the invariant distribution of $\tilde{z} = (\tilde{a}, \tilde{x}, j, \epsilon, s)$ in the stationary equilibrium, where the sum of all individuals is normalized to one.

In stationary equilibrium, the following conditions hold:

1. Households maximize their intertemporal utility as described in Section Appendix B.4, implying the policy functions $\tilde{c}(.)$ and $\tilde{a}'(.)$ of both types, the labor supply $l(.)$ of the workers, and the investment $\tilde{k}(.)$ of the entrepreneurs.
2. In a factor market equilibrium, factors are rewarded with their marginal product presented by (7).
3. Total production $\tilde{Y}_t$ is equal to the sum of production in both sectors, $\tilde{Y}_{c,t}$ and $\tilde{Y}_{e,t}$:
$$\tilde{Y}_t = \tilde{Y}_{c,t} + \tilde{Y}_{e,t}.$$
4. The government budget (8) is financed by debt in every period t:
$$\tilde{T}r_t + \tilde{G}_t + r_t\tilde{D}_t = \tilde{T}ax_t + (1+n)(1+g)\tilde{D}_{t+1} - \tilde{D}_t + \tilde{B}eq_t.$$

5. The budget of the social security authority is balanced in every period t:

$$\tilde{P}_t = \tau^p \tilde{w}_t L_{c,t}.$$

6. The distribution $\tilde{\mu}(\tilde{a}, \tilde{x}, j, \epsilon, s)$ is invariant.
7. Individual and aggregate behaviors are consistent (we drop the index for period t in the following):

$$\tilde{Y}_e = \sum_{s=1}^{45} \sum_j \int_{\tilde{a}} \int_{\tilde{x}} \theta^{e,s,j} \left(\tilde{k}(\tilde{a}, \tilde{x}, j, e, s)\right)^{\alpha} \left(\bar{l}\right)^{1-\alpha} \; \tilde{\mu}(\tilde{a}, \tilde{x}, j, \epsilon, s) \; d\tilde{a} \; d\tilde{x}, \tag{B.5a}$$

$$\tilde{P} = \sum_{s=46}^{80} \sum_j \sum_{\epsilon} \int_{\tilde{a}} \int_{\tilde{x}} \tilde{p}(\tilde{x}) \; \tilde{\mu}(\tilde{a}, \tilde{x}, j, \epsilon, s) \; d\tilde{a} \; d\tilde{x}, \tag{B.5b}$$

$$\begin{aligned}(1+g)(1+n)\tilde{B}eq' = & \sum_{s=1}^{80} \sum_j \sum_{\epsilon} \int_{\tilde{a}} \int_{\tilde{x}} (1-\phi_s)\left(1+(1-\tau_r)r\right) \\ & \times \tilde{a}'(\tilde{a}, \tilde{x}, j, \epsilon, s) \; \tilde{\mu}(\tilde{a}, \tilde{x}, j, \epsilon, s) \; d\tilde{a} \; d\tilde{x},\end{aligned} \tag{B.5c}$$

$$\tilde{T}r = \tilde{t}r, \tag{B.5d}$$

$$\tilde{\Omega} = \sum_{s=1}^{80} \sum_j \sum_{\epsilon} \int_{\tilde{a}} \int_{\tilde{x}} \tilde{a} \; \tilde{\mu}(\tilde{a}, \tilde{x}, j, \epsilon, s) \; d\tilde{a} \; d\tilde{x}, \tag{B.5e}$$

$$\tilde{C} = \sum_{s=1}^{80} \sum_j \sum_{\epsilon} \int_{\tilde{a}} \int_{\tilde{x}} \tilde{c}(\tilde{a}, \tilde{x}, j, \epsilon, s) \; \tilde{\mu}(\tilde{a}, \tilde{x}, j, \epsilon, s) \; d\tilde{a} \; d\tilde{x}, \tag{B.5f}$$

$$\tilde{L}_e = \sum_{s=1}^{45} \sum_j \int_{\tilde{a}} \int_{\tilde{x}} \bar{l} \; \tilde{\mu}(\tilde{a}, \tilde{x}, j, e, s) \; d\tilde{a} \; d\tilde{x}, \tag{B.5g}$$

$$\tilde{L}_c = \sum_{s=1}^{45} \sum_j \int_{\tilde{a}} \int_{\tilde{x}} l(\tilde{a}, \tilde{x}, j, w, s) \; \tilde{\mu}(\tilde{a}, \tilde{x}, j, w, s) \; d\tilde{a} \; d\tilde{x}, \tag{B.5h}$$

$$\tilde{K}_e = \sum_{s=1}^{45} \sum_j \int_{\tilde{a}} \int_{\tilde{x}} \tilde{k}(\tilde{a}, \tilde{x}, j, e, s) \; \tilde{\mu}(\tilde{a}, \tilde{x}, j, e, s) \; d\tilde{a} \; d\tilde{x}, \tag{B.5i}$$

$$\tilde{K} = \tilde{K}_e + \tilde{K}_c, \tag{B.5j}$$

$$\tilde{L} = \tilde{L}_e + \tilde{L}_c, \tag{B.5k}$$

$$\tilde{T}ax = \tau_r r \tilde{\Omega} + \tau^c \tilde{C} + \sum_{s=1}^{80} \sum_j \int_{\tilde{a}} \int_{\tilde{x}} \tilde{T}\left(\tilde{y}(\tilde{a}, \tilde{x}, j, e, s)\right) \; \tilde{\mu}(\tilde{a}, \tilde{x}, j, e, s) \; d\tilde{a} \; d\tilde{x}, \tag{B.5l}$$

where $\tilde{y}$ denotes the stationary non-interest income of the household with the individual state variable $\tilde{z}$.

8. The capital market clears: total household savings are equal to the sum of the total capital employed in the corporate and the entrepreneurial sectors plus government debt:

$$\tilde{\Omega} = \tilde{K}_c + \tilde{K}_e + \tilde{D}.$$

9. The labor market clears such that $\tilde{L}_c$ and $\tilde{L}_e$ are given by (B.5h) and (B.5g), respectively.
10. Profits in the credit sector are zero.
11. The goods market clears such that

$$\tilde{Y} = \tilde{C} + \tilde{G} + (g + n + gn + \delta)\tilde{K}.$$

Appendix B.6. Calibration

The calibration of the parameters follows the description in Section 3.7 and Table 4, with the following exceptions. First, we do not use a linear labor tax rate τ^l but choose income tax parameters $\zeta_1 = 0.272051$ in our tax function (12). The estimate is taken from Holter et al. (2019) for the case of Germany (married with two children). The parameter $\zeta_0 = 0.0843$ is set to match the labor income tax rate of the average income $\bar{y}$, as in our benchmark model (equal to τ^l). The pensions are calibrated so that we model the German gross replacement rate *repl* as presented in Table 4 with respect to an average labor supply of 0.3. Finally, we apply the estimate of the transition matrix P provided in (1) for our value of $\Pi(\epsilon', j'|\epsilon, j)$. We calibrate the initial mass of the entrepreneurs equal to 11.3% so that the transition matrix P implies an average share of the entrepreneurs in the labor force equal to 8.0%.

Appendix B.7. Computation

The individual policy functions are computed with the help of value function iteration. We use a nested golden section search to optimize over the two continuous individual state variables, wealth $\tilde{a}$ and cumulated contributions $\tilde{x}$. The stationary distribution of the individual state variables is computed with the method described in Algorithm 7.2.3 in Heer and Maußner (2009). The update to the aggregate state variables uses a simple dampening method as in Judd (1998).