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A comparison of high-frequency realized variance measures: Duration- vs. return-based approaches

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A comparison of high-frequency realized variance measures:
Duration- vs. return-based approaches

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Abstract

We study the accuracy of a variety of parametric price duration-based realized variance estimators constructed via various financial duration models and compare their forecasting performance with the performance of various non-parametric return-based realized variance estimators. Our financial duration models consist of an ACD(1,1), its logarithmic version, Log-ACD(1,1), and its long-memory version, FIACD(1,1), as well as the Markov-switching multifractal duration (MSMD) model and the factorial hidden Markov duration (FHMD) process. In an empirical study using high-frequency data on ten stocks traded on the New York Stock Exchange (NYSE), our in- and out-of-sample results show that the parametric price duration-based realized variance (RV) estimators, especially the ACD-based RV estimator, perform better than the non-parametric return-based RV estimators. Furthermore, we also find that the price duration-based and return-based RV models produce more accurate and valid Value-at-Risk forecasts than the GARCH(1,1) model.

Keywords: High-frequency data; Price duration; Realized measures of integrated variance; Value-at-Risk.

JEL classification: C41, C52, C53, C58.

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1 Introduction

Although 60 years of research have been dedicated to the study of asset price variability, it has lost none of its appeal for academia and practitioners, due to its ongoing importance for risk management, derivative pricing and asset allocation. Initially, a lot of attention was devoted to the development of parametric approaches to volatility modeling. The incorporation of the stylized facts of financial return time series data, such as volatility clustering, leverage, fat tails, jumps, long memory etc., has led to an abundance of GARCH-type and stochastic volatility (SV) models, see Zaharieva et al. (2020) and the references therein for a recent overview.

Owing to the advent of high-frequency data, the focus for the measurement of asset price variability has been shifted from parametric to non-parametric methods during the last two decades. The first model-free approaches of daily variance estimators utilizing high-frequency intra-day return data, date back to the seminal works of Andersen et al. (2001a, 2001b), and Barndorff-Nielsen and Shephard (2002a, 2002b). They established a realized variance (RV) estimator to obtain a consistent estimate of an asset's integrated variance. However, exploiting the entire record of observations induces a severe bias in the proposed RV estimator, when the price process is distorted by market microstructure noise (cf., *inter alia*, Hansen and Lunde, 2006; Bandi and Russell, 2008). As a consequence, a variety of modifications and refinements have been subsequently proposed to handle the contamination of the observed (transaction) prices due to market imperfections. The subsampling technique of Zhang et al. (2005) seeks to increase the sample size without increasing the sampling frequency, by combining realized variance estimates that are computed using different sparse subsamples of the same time scale. Zhang et al. (2005) and Zhang (2006) apply this idea to subsamples of different time scales, which leads to the two-scale and multi-scale RV estimators. Another way to construct a consistent estimator can be achieved by weighting the original returns, e.g. by employing kernel functions as in Barndorff-Nielsen et al. (2008), or by pre-averaging the returns as in Podolskij and Vetter (2009), Jacod et al. (2009) and Christensen et al. (2014). The (jump-robust) realized bipower variation estimator of Barndorff-Nielsen and Shephard (2006) is a further remarkable contribution to this field. For a more comprehensive review on RV estimators, see the extensive study of Liu et al. (2015).

The fact that high-frequency data has become omnipresent has also fuelled the emergence of another research area. The last two decades have witnessed a huge effort devoted to modeling high-frequency financial duration data, i.e. the time that has passed between two financial events. The vast majority of studies focus on price durations, which are defined as the time required for a (cumulative) change in the price beyond a pre-specified threshold. The reasoning is twofold: (i) to test some market microstructure theories and to understand the abundance of issues related to trading and price-adjustment processes (cf. surveys of Madhavan, 2000; Biais et al., 2005; Hasbrouck, 2007) and (ii) because modeling financial durations may help in accurately predicting instantaneous volatility, which is highly beneficial to measuring and managing intra-day exposure to risk (cf., *inter alia*, Giot 2005; Dionne et al., 2009; Liu and Tse, 2015).

Inspired by the GARCH model, Engle and Russell (1998) first propose an autoregressive conditional duration (ACD) model for analyzing financial durations. Analogously to the development of the GARCH model family, various extensions of the standard ACD model have been introduced in the literature in order to appropriately reproduce the stylized facts of financial durations, see Pacurar (2008) for a detailed literature review on ACD models.

Moreover, the pioneering work of Engle and Russell (1998) can also be regarded as the starting point that unifies the modeling of (price) durations with the concept of realized variance. Based on their established linkage between the conditional hazard function and instantaneous volatility Tse and Yang (2012) and Hong et al. (2023) introduce a parametric price duration-based approach to estimate the intra-day variance. Their duration-based method offers two advantages over the return-based approaches: (i) as noted by Andersen et al. (2009) it exhibits robustness to jumps and market microstructure noise and (ii) utilizes the data more efficiently. When constructing price duration data, sampling occurs more frequently in turmoil periods (with many price changes) than in tranquil periods (with few price changes), whereas the sampling mechanism for intra-day returns completely ignores current market dynamics and only records prices in fixed time intervals. Hence, the sampling of price duration can extract the data's informational content more efficiently and preserves the irregular spacing feature of the raw data qua construction.

Despite the promising results of the price duration-based approaches, the current state of the literature is still at an early stage. It lacks a systematic comparison of the impact of different modeling frameworks governing the (price) durations, sampling methods and distributional assumptions. To this end, our paper seeks to close this gap, and considers different model setups and investigates their impact on estimation accuracy and forecasting performance for daily variance.

For our study, we employ the Markov-switching multifractal duration (MSMD) process. The successful application of the Markov-switching multifractal (MSM) model of Calvet and Fisher (2004) in modeling and forecasting financial market volatility motivated both Chen et al. (2013) and Žikeš et al. (2017) to adapt the model to the duration setting. The MSMD model can reproduce, by its very principle of construction, short- and long-memory as observed in financial duration data.

Recently, Schulte-Tillmann and Segnon (2024) applied the factorial hidden Markov volatility (FHMV) process of Augustyniak et al. (2019) to the framework of durations. The resulting factorial hidden Markov duration (FHMD) model can be regarded as a viable alternative to the MSMD process and traditional ACD-type models. By design it possesses a very flexible autocorrelation structure capable of reproducing a wide range of persistence observed in financial duration data. Moreover, it also embeds a jump component and more versatile support than the MSMD process, and can thus generate richer dynamics. The hierarchical structure of the latent components in the FHMD process may enable the model to reproduce self-similarity properties observed in financial durations. The presence of self-similarity in financial durations suggests that the information flow arrives in the markets not only in clusters, but also in cascades. This is in line with the conjecture of heterogeneous market participants who act on different time scales, and have limited attention.

To complete our set of competitor models we consider the ACD(1,1) process (Engle and Russell, 1998) as the benchmark model, its logarithmic version, the Log-ACD(1,1) model (Bauwens and Giot, 2000) and the FIACD(1,1) process (Jasiak, 1999) as a genuine long-memory model.

We apply the models to price durations of ten actively traded stocks on the New York Stock Exchange (NYSE) to conduct an in-sample and an out-of-sample analysis. Moreover, we also compare their performance with those of well-established RV estimators. In an application to forecast the stocks' Value-at-Risk (VaR) for a holding period of one day ahead, we find that predictions based on intra-day duration data are superior compared to those that rely on daily and intra-day return data.

The rest of this paper is organized as follows. Section 2 presents the theoretical framework of price duration-based variance estimation, describes the duration models and the different RV estimators. Section 3 discusses the empirical application and Section 4 concludes.

2 Theoretical foundation

We consider an arbitrage-free financial market, in which an asset's (continuous-time) log-price process follows a semimartingale (cf. Back, 1991; Delbaen and Schachermayer, 1994) of the form

$$dX(t) = \mu(t)dt + \sigma(t)dW(t), \tag{1}$$

where $W(t)$ is a standard Brownian motion, $\mu(t)$ is a continuous drift process with locally bounded variation and $\sigma(t)$ denotes a Càdlàg volatility process. The riskiness of such an asset over the time span $[0, t]$ can be reflected by its integrated variance,

$$IV_t = \int_0^t \sigma^2(s)ds. \tag{2}$$

Starting at the beginning of a trading period, t_0, we mark each point in time, at which the absolute cumulative price change exceeds a certain threshold, ι, by $t_1 < t_2 < \ldots < t_N$. As a result, we obtain the strictly increasing sequence of hitting times $\{t_j\}_{j=0,\ldots,N}$. Moreover, we denote the associated total number of hitting events up to time t by N_t. In our analysis, we concentrate on the time difference between consecutive arrivals, and refer to them as price durations, $d_i = t_i - t_{i-1}$, to infer the integrated variance, our objective of interest.

2.1 Financial duration models

In general, financial durations, d_i, can be modeled as

$$d_i = \psi_i \epsilon_i, \qquad i \in \mathbb{Z}, \tag{3}$$

where $\{\epsilon_i\}$ is a sequence of independent and identically distributed (*i.i.d.*) unit-mean innovations with positive support. We then present various processes that model the dynamics of $\{\psi_i\}$ in different ways. However, all models are estimated by maximum likelihood.

2.2 Autoregressive conditional duration (ACD) models

2.2.1 ACD(1,1) model

In the ACD(1,1) framework proposed by Engle and Russell (1998), the conditional duration process is given by

$$\psi_i = \omega + \alpha_1 d_{i-1} + \beta_1 \psi_{i-1}. \tag{4}$$

The conditional duration, ψ_i, follows an autoregressive process à la GARCH. To ensure the stationarity and positivity of the conditional duration, the parameters in the model have to satisfy $\omega > 0$, $\alpha_1, \beta_1 \geq 0$ and $\alpha_1 + \beta_1 < 1$.

2.2.2 Log-ACD(1,1) model

To avoid the non-negativity restrictions in the original ACD model and to provide more flexibility, Bauwens and Giot (2000) propose the Log-ACD specification that models the conditional duration, ψ_i, as follows:

$$\varphi_i = \omega + \alpha\epsilon_{i-1} + \beta\varphi_{i-1}, \tag{5}$$

where φ_i is the logarithm of the conditional duration, i.e. $\psi_i = \exp(\varphi_i)$. This specification is known in the literature as the Log-ACD model of type 2 (Hautsch, 2012).

2.2.3 Fractionally integrated ACD$(1, d, 1)$ model

In order to accurately reproduce the long-range dependence that characterizes the autocorrelation function of financial duration data, Jasiak (1999) proposes the FIACD$(1, d, 1)$ framework that expresses the conditional duration as

$$\begin{aligned} \psi_i &= \omega \left(1 - \beta_1 L\right)^{-1} + \left[1 - \left(1 - \beta_1 L\right)^{-1} \left(1 - \phi_1 L\right) \left(1 - L\right)^d\right] d_i \\ &= \varpi + B(\mathrm{L}) d_i, \end{aligned} \tag{6}$$

where $\phi_1 = \alpha_1 + \beta_1$, $B(\mathrm{L}) = b_1\mathrm{L} + b_2\mathrm{L}^2 + \ldots = 1 - (1 - \beta_1\mathrm{L})^{-1}(1 - \phi_1\mathrm{L})(1 - \mathrm{L})^d$ is a lag polynomial of infinite order with $b_k \geq 0$, $k = 1, 2, \ldots$ and $\varpi = \omega\left(1 - \beta_1 L\right)^{-1} > 0$.

The parameters, b_k, can be expressed as

$$
\begin{aligned}
b_1 &= \phi_1 - \beta_1 + d \\
b_2 &= (d - \beta_1)(\beta_1 - \phi_1) + \frac{d(1-d)}{2} \\
&\vdots \\
b_k &= \beta_1 b_{k-1} + \left(\frac{k-1-d}{k} - \phi_1 \right) \pi_{d,k-1},
\end{aligned}
\tag{7}
$$

where $\pi_{d,k} = \pi_{d,k-1}(k-1-d)k^{-1}$ for $k = 2, 3, \ldots$. Note that $\pi_{d,k}$ represents the terms of the expansion of $(1-L)^d$, which can be expressed as

$$
\pi_d(L) = \sum_{k=0}^{\infty} \pi_{d,k} L^k. \tag{8}
$$

To ensure positivity of the conditional durations in the FIACD$(1, d, 1)$ framework, the parameters ϕ_1, β_1 and d must fulfill the following conditions, see Bollerslev and Mikkelsen (1996):

$$
\beta_1 - d \leq \phi_1 \leq \frac{2-d}{3}, \ \ d\left(\phi_1 - \frac{1-d}{2} \right) \leq \beta_1 (d - \beta_1 + \phi_1). \tag{9}
$$

As stressed in Jasiak (1999), the FIACD$(1, d, 1)$ can easily be estimated via the maximum likelihood method by choosing a suitable truncation point that we set to 1,000 in our empirical study.

Remark. *The necessary and sufficient conditions for the covariance stationarity of the ACD*$(1,1)$ *are provided in Engle and Russell (1998). Furthermore, ergodicity, mixing and the existence of moments are discussed in detail in Meitz and Saikkonen (2011). We note that the FIACD*$(1, d, 1)$ *is not covariance stationary, but strictly stationary, see Jasiak (1999). We refer the reader to Conrad and Haag (2006) for less restrictive positivity conditions than those provided in Eq.* (9).

2.3 Hidden Markov duration (HMD) models

2.3.1 MSMD model

The adaption of the MSM stochastic volatility model of Calvet and Fisher (2001, 2004) to the duration setting has been proposed independently by Chen et al. (2013) and Žikeš et al. (2017). In the resulting MSMD framework, the price durations are defined as in Eq. (3).

More specifically, the process $\{\psi_i\}$ is latent and composed of k_v independent components, $V_i^{(j)}, j = 1, \ldots, k_v$, that are multiplicatively connected and scaled with factor $\bar{\psi}$, i.e.

$$\psi_i = \bar{\psi} \prod_{j=1}^{k_v} V_i^{(j)}. \tag{10}$$

The dynamics of the process result from the renewal mechanism underlying the components. At time i, each component, $V_i^{(j)}$, is either renewed with probability γ_j or remains unchanged at its previous value with probability $(1 - \gamma_j)$. In the event of a renewal, a new value from a discrete distribution V with support $\{v_0, 2 - v_0\}$, where $v_0 \in (1, 2)$, is drawn with equal probability. Hence, each component is a unit-mean, two-state Markov chain that can be characterized by

$$V_i^{(j)} = \begin{cases} V_{i-1}^{(j)}, & \text{with prob. } 1 - \gamma_j, \\ v_0, & \text{with prob. } 0.5\gamma_j, \\ 2 - v_0, & \text{with prob. } 0.5\gamma_j \end{cases} \tag{11}$$

with corresponding transition probability matrix

$$\mathbf{P}_j = \begin{pmatrix} 1 - 0.5\gamma_j & 0.5\gamma_j \\ 0.5\gamma_j & 1 - 0.5\gamma_j \end{pmatrix}. \tag{12}$$

The parametrization of the transition probability,

$$\gamma_j = 1 - (1 - \gamma_1)^{b^{j-1}}, \tag{13}$$

where $\gamma_1 \in (0, 1)$ and $b > 1$ for $j = 1, \ldots, k_v$, leads to a multiplier renewal with different frequencies.

2.3.2 FHMD model

In the FHMD framework of Schulte-Tillmann and Segnon (2024), the process $\{\psi_i\}$ is also latent and can be formalized as a product of several independent processes, $\{M_i\}$ and $\{C_i^{(j)}\}$, $j = 1, \ldots, k_c$, i.e.

$$\psi_i = \bar{\psi} M_i \left(c_0 \prod_{j=1}^{k_c} C_i^{(j)} \right). \tag{14}$$

Apart from the parameter $\bar{\psi}$, which also serves as a scaling factor as in the MSMD model, the structure of the other components, differ. The process $\{M_i\}$ is a sequence of *i.i.d.* discrete random variables satisfying $\mathbb{E}(M_i) = 1$ and each process $\{C_i^{(j)}\}$, $j = 1, \ldots, k_c$, is a Markov chain with unique support, that is $\operatorname{supp}(C_i^{(j)}) = \{c_j, 1\}$. The recursive definition of the outcome c_j,

$$c_j = 1 + \theta_c^{j-1}(c_1 - 1), \quad j = 2, \ldots, k_c, \tag{15}$$

where $c_1 > 1$ and $\theta_c \in (0, 1)$, implies a hierarchical structure in the individual supports, such that $c_1 > c_2 > \ldots > c_{k_c} > 1$. In contrast to the MSMD model, the switching behavior for all two-state Markov chains, $\{C_i^{(j)}\}$, is governed by the same transition probability matrix,

$$\mathbf{P} = \begin{pmatrix} p & 1-p \\ 1-p & p \end{pmatrix}, \tag{16}$$

where $p \in (0, 1)$. Moreover, by setting $c_0 = 1/\mathbb{E}\left(\prod_{j=1}^{k_c} C_i^{(j)}\right)$, we ensure that $\mathbb{E}\left(c_0 \prod_{j=1}^{k_c} C_i^{(j)}\right) = 1$. To facilitate maximum likelihood estimation of this hidden Markov model, we stack the single components into the state vector $C_i = \left(C_i^{(1)}, \ldots, C_i^{(k_c)}\right)$ with state space

$$\Delta_C = \{c_1, 1\} \times \{c_2, 1\} \times \cdots \times \{c_{k_c}, 1\}. \tag{17}$$

The component $\{M_i\}$ follows an *i.i.d.* discrete random process with probability function

$$\Pr(M_i = m_0 \cdot m_i) = \begin{cases} q(k_m - 1)^{-1}, & \text{for} \quad i = 1, \ldots, k_m - 1, \\ 1 - q, & \text{for} \quad i = k_m \end{cases}, \tag{18}$$

where $q \in (0, 1)$, $m_i = 1 + \theta_m^{i-1}(m_1 - 1)$ with $m_1 > 1$, $m_{k_m} = 1$ and $\theta_m \in (0, 1)$ for $i = 2, \ldots, k_m - 1$. Analogous to the above definition, the recursive specification implies a hierarchical structure in the magnitude of the possible outcomes by $m_1 > m_2 > \ldots > m_{k_m} = 1$ and setting $m_0 = \left[1 + q\frac{(m_1-1)\left(1-\theta_m^{k_m-1}\right)}{(k_m-1)(1-\theta_m)}\right]^{-1}$, results in the normalization of the distribution of the component, such that $\mathbb{E}(M_i) = 1$. Accordingly, the process $\{M_i\}$ is non-persistent and takes on values of the finite state space $\Delta_M = \{m_0 m_1, m_0 m_2, \ldots, m_0 m_{k_m}\}$, which enables it to capture abrupt spikes in the durations.

Combining the Markov chain $\{C_i\}$ and the process $\{M_i\}$ leads to the overall state vector $\{\widetilde{\psi}_i\}$ with state space $\Delta_{\widetilde{\psi}} = \Delta_C \otimes \Delta_M$. Due to the distinct support of each component $\{C_i^{(j)}\}$ and the additional jump component, the FHMD framework is capable of reflecting richer dynamics than the MSMD model (cf. Section 2.2 in Schulte-Tillmann and Segnon, 2024).

Throughout this article, we let the innovations follow a Burr distribution to ensure a flexible shape of the conditional intensity function or the standard exponential distribution as the baseline specification. Following the notation of Hautsch (2012), the Burr probability density function and cumulative density function are given by

$$f(\epsilon|\lambda,\eta,a) = \frac{a}{\lambda}\left(\frac{\epsilon}{\lambda}\right)^{a-1}\left[1+\eta\left(\frac{\epsilon}{\lambda}\right)^{a}\right]^{-(1+\eta^{-1})}, \quad \epsilon>0, \lambda>0, a>0, \eta>0 \tag{19}$$

and

$$F(\epsilon|\lambda,\eta,a) = 1-\left[1+\eta\left(\frac{\epsilon}{\lambda}\right)^{a}\right]^{-\eta^{-1}}. \tag{20}$$

Note that the Burr distribution nests the Weibull distribution for $\eta \to 0$, and by additionally stipulating that $a = 1$, it also nests the exponential distribution. We set

$$\lambda = \eta^{1+a^{-1}} \cdot B(1+a^{-1}, \eta^{-1}-a^{-1})^{-1},$$

to scale the mean of the innovations to unity.[1]

2.4 Price duration-based estimation of variance

Of central importance for the inference of intra-day variance from the domain of price durations is the conditional intensity function. It represents the probability of a price event at time $t > t_{i-1}$ given the price event has not occurred before time t and the information set available up to arrival time t_{i-1}, denoted by $\mathcal{F}_{i-1}$:

$$\lambda(d|\mathcal{F}_{i-1}) = \frac{f(d|\mathcal{F}_{i-1})}{1-F(d|\mathcal{F}_{i-1})}, \tag{21}$$

where $d = t - t_{i-1}$ and $f(\cdot)$ and $F(\cdot)$ denote the density and cumulative distribution function, respectively. According to Engle and Russell (1998), the conditional intensity function (or equivalently, the conditional hazard function) can be linked to the conditional instantaneous variance by

$$\sigma^2(t|\mathcal{F}_{i-1}) = \left(\frac{\iota}{P_{i-1}}\right)^2 \lambda(d|\mathcal{F}_{i-1}), \tag{22}$$

where P_{i-1} denotes the asset's price at time t_{i-1}. Building on this work, Tse and Yang (2012) and Hong et al. (2023) provide the foundation for non-parametric as well as parametric price

[1] $B(\cdot,\cdot)$ denotes the Beta function, i.e. $B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$. Moreover, we assume $a > \eta$ to ensure the existence of the first moment.

duration-based estimators. By integrating over the interval (t_{i-1}, t_i), we obtain an estimator of the integrated variance. However, we are generally interested in the integrated variance over a specific time interval $[t_0, t_N]$, which represents a trading day, an hour etc. Hence, we aggregate the single integrals as follows

$$\begin{aligned}\widehat{IV}_t &= \sum_{i=1}^{N} \int_{t_{i-1}}^{t_i} \sigma^2(t|\mathcal{F}_{i-1})dt \\ &= \sum_{i=1}^{N} \left(\frac{\iota}{P_{i-1}}\right)^2 \int_{t_{i-1}}^{t_i} \lambda(t - t_{i-1}|\mathcal{F}_{i-1})dt \\ &= -\iota^2 \sum_{i=1}^{N} \frac{\ln\left(1 - F(d_i|\mathcal{F}_{i-1})\right)}{P_{i-1}^2}. \end{aligned} \tag{23}$$

Given Eq. (23) non-parametric and parametric approaches can be employed for the estimation of the integrated variance. As Hong et al. (2023) demonstrate, replacing the summands in Eq. (23) by their expectations leads to a non-parametric variance estimator (NPDV), which is given by

$$\text{NPDV} = \sum_{i=1}^{N} \left(\frac{\iota}{P_{i-1}}\right)^2. \tag{24}$$

Our focus, though, is on the parametric approach. We employ the models described in Sections 2.2 and 2.3 in order to obtain parametric price duration-based variance estimators (PDV).

2.5 HMD-based variance estimator

Since Tse and Yang (2012) and Hong et al. (2023) show how price duration-based variance estimators can be obtained based on ACD-type models, we concentrate on hidden Markov model frameworks. To this end, let $f(d_i|\widetilde{\psi}_i, \mathcal{F}_{i-1})$ denote the conditional density given state $\widetilde{\psi}_i$ and information set $\mathcal{F}_{i-1}$.[2] Then, we obtain $f(d_i|\mathcal{F}_{i-1})$ by integrating out the states from the joint density, $f(d_i, \widetilde{\psi}_i|\mathcal{F}_{i-1})$, as follows

$$f(d_i|\mathcal{F}_{i-1}) = \sum_{\widetilde{\psi}_i \in \Delta_{\widetilde{\psi}}} f(d_i|\widetilde{\psi}_i, \mathcal{F}_{i-1}) f(\widetilde{\psi}_i|\mathcal{F}_{i-1}). \tag{25}$$

[2] For the sake of generality, let $\widetilde{\psi}_i$ also denote the state vector in the MSMD model.

As a consequence, the corresponding cumulative probability function is given by

$$\begin{aligned} F(d_i|\mathcal{F}_{i-1}) &= \int_0^{d_i} f(u_i|\mathcal{F}_{i-1}) du_i \\ &= \int_0^{d_i} \sum_{\widetilde{\psi}_i \in \Delta_{\widetilde{\psi}}} f(u_i|\widetilde{\psi}_i, \mathcal{F}_{i-1}) f(\widetilde{\psi}_i|\mathcal{F}_{i-1}) du_i \\ &= \sum_{\widetilde{\psi}_i \in \Delta_{\widetilde{\psi}}} f(\widetilde{\psi}_i|\mathcal{F}_{i-1}) \int_0^{d_i} f(u_i|\widetilde{\psi}_i, \mathcal{F}_{i-1}) du_i. \end{aligned} \tag{26}$$

Given that the innovations are Burr distributed, the conditional intensity function, $\lambda(d_i|\mathcal{F}_{i-1})$, becomes

$$\lambda(d_i|\mathcal{F}_{i-1}) = \frac{\sum_{\widetilde{\psi}_i \in \Delta_{\widetilde{\psi}}} \frac{a}{\psi_i} c(a,\eta) \cdot \left(\frac{d_i}{\psi_i} c(a,\eta)\right)^{a-1} \left[1 + \eta \left(\frac{d_i}{\psi_i} c(a,\eta)\right)^a\right]^{-(1+\eta^{-1})} f(\widetilde{\psi}_i|\mathcal{F}_{i-1})}{\sum_{\widetilde{\psi}_i \in \Delta_{\widetilde{\psi}}} \left(1 + \eta \left(\frac{d_i}{\psi_i} c(a,\eta)\right)^a\right)^{-\eta^{-1}} f(\widetilde{\psi}_i|\mathcal{F}_{i-1})}, \tag{27}$$

where $c(a,\eta) = B(1 + a^{-1}, \eta^{-1} - a^{-1}) \cdot \eta^{-\left(1+a^{-1}\right)}$.[3] For standard exponentially distributed innovations, the conditional intensity function simplifies to

$$\lambda(d_i|\mathcal{F}_{i-1}) = \frac{\sum_{\widetilde{\psi}_i \in \Delta_{\widetilde{\psi}}} \frac{1}{\psi_i} \exp\left(-\frac{d_i}{\psi_i}\right) f(\widetilde{\psi}_i|\mathcal{F}_{i-1})}{\sum_{\widetilde{\psi}_i \in \Delta_{\widetilde{\psi}}} \exp\left(-\frac{d_i}{\psi_i}\right) f(\widetilde{\psi}_i|\mathcal{F}_{i-1})}. \tag{28}$$

Combining Eqs. (22) and (25) then results in the parametric price duration-based variance estimator:

$$\text{PDV}_{\text{HMD-Burr}} = -\iota^2 \sum_{i=1}^{N} \ln \left(\sum_{\widetilde{\psi}_i \in \Delta_{\widetilde{\psi}}} \left(1 + \left(\frac{d_i}{\psi_i} \cdot \frac{B\left(1 + a^{-1}, \eta^{-1} - a^{-1}\right)}{\eta}\right)^a\right)^{-\eta^{-1}} f(\widetilde{\psi}_i|\mathcal{F}_{i-1}) \right) / P_{i-1}^2 \tag{29}$$

We receive the conditional density function $f(\widetilde{\psi}_i|\mathcal{F}_{i-1})$ as a by-product of the maximum likelihood estimation process, which is described thoroughly in Schulte-Tillmann and Segnon (2024). Moreover, note that we obtain the price duration-based variance estimator with an

[3] Note that ψ_i is completely determined by the state variable $\widetilde{\psi}_i$.

underlying exponential distribution as a special case, when $\eta \to 0$ and $a = 1$:

$$\text{PDV}_{\text{HMD-Exp}} = -\iota^2 \sum_{i=1}^{N} \ln \left(\sum_{\widetilde{\psi}_i \in \Delta_{\widetilde{\psi}}} \exp\left(-\frac{d_i}{\psi_i}\right) f(\widetilde{\psi}_i | \mathcal{F}_{i-1}) \right) / P_{i-1}^2. \tag{30}$$

2.6 Review of RV methods

Before we briefly review selected non-parametric and return-based estimation approaches, we introduce some notation. The entire record of observed (log-)transaction prices on a trading day is given by $\{X_{t_0}, X_{t_1}, \ldots, X_{t_n}\}$. Employing the complete set of returns, though, will severely distort the estimation due to microstructure noise. Thus, we consider a sparse sample $\{X_{t_0}, X_{t_{1\Delta}}, X_{t_{2\Delta}}, \ldots, X_{t_{(n_\Delta - 1)\Delta}}, X_{t_{n_\Delta \Delta}}\}$ with reduced sample size n_Δ, that we obtain by sampling with interval Δ.

The basic RV estimator is then defined by

$$\text{RV}^{\Delta} = \sum_{i=1}^{n_\Delta} (X_{t_{i\Delta}} - X_{t_{(i-1)\Delta}})^2 = \sum_{i=1}^{n_\Delta} r_{t_{i\Delta}}^2. \tag{31}$$

In our empirical application, we use a sampling interval of 5 minutes. However, by doing so, we discard large quantities of data. A straightforward strategy for making use of more observations without being exposed to severe estimation bias is the subsampling technique proposed by Zhang et al. (2005). To illustrate the idea, suppose that the entire grid of observation times is given by $\mathcal{G} = \{t_0, \ldots, t_n\}$. By partitioning the original grid into K_Δ subsamples, $\mathcal{G}^{(k_\Delta)}$, for $k_\Delta = 1, \ldots, K_\Delta$, with sampling interval Δ, we can compute the realized variance for each subsample

$$\text{RV}^{(k_\Delta)} = \sum_{t_i \in \mathcal{G}^{(k_\Delta)}} r_{t_i}^2. \tag{32}$$

Note that all subsamples share the same sampling interval, i.e. they exhibit the same time scale, but the subsamples differ in their starting point, so that they are non-overlapping. By taking the average, we obtain the subsampled version of the basic realized variance estimator

$$\text{RV}_{\text{ss}}^{\Delta} = \frac{1}{K_\Delta} \sum_{k_\Delta=1}^{K_\Delta} \text{RV}^{(k_\Delta)}. \tag{33}$$

The above proposed estimator, however, only reduces the distortion induced by microstructure noise, but does not eliminate the bias. Following Zhang et al. (2005), a biascorrected version can be obtained by combining two different time scales of the basic realized variance estimator which is computed on the basis of all observations and its subsampled version,

$$\text{TSRV}^{\Delta} = \text{RV}_{\text{ss}}^{\Delta} - \frac{\bar{n}_{K_\Delta}}{n}\text{RV}, \text{ where } \bar{n}_{K_\Delta} = \frac{1}{K_\Delta}\sum_{k_\Delta=1}^{K_\Delta} n_{k_\Delta} \text{ and } \text{RV} = \sum_{i=1}^{n} r_{t_i}^2. \tag{34}$$

As recommended by the authors, we sample the price process every 5 seconds in our application.

Zhang's (2006) generalization of this approach to multiple time scales (i.e. utilizing different sampling time intervals Δ_i for $i = 1, \ldots, M$) yields a more efficient estimator, which is also unbiased in the presence of microstructure noise, that is

$$\text{MSRV} = \sum_{i=1}^{M} \alpha_i \text{RV}^{k_{\Delta_i}} + \frac{1}{n}\text{RV}, \tag{35}$$

where the weights α_i are appropriately chosen.

Barndorff-Nielsen et al. (2008) propose the realized kernel estimator, which is defined as

$$\text{RK} = \gamma_0 + \sum_{h=1}^{H} k\left(\frac{h-1}{H}\right)(\gamma_h + \gamma_{-h}), \tag{36}$$

where $k(x)$ for $x \in [0,1]$ represents a weighting function and the h-th realized autocovariance can be computed by

$$\gamma_h = \sum_{i=|h|+1}^{n} r_{t_i} r_{t_{i-h}} \quad \text{for } h \in \mathbb{Z} \text{ and } |h| < H. \tag{37}$$

We employ several weighting functions: (i) the Parzen kernel, (ii) the cubic kernel, (iii) the Bartlett kernel and (iv) the (modified) Turkey-Hanning kernel (abbreviated by '*th2*'). The optimal bandwidth length H is chosen according to Barndorff-Nielsen et al. (2009). Moreover, we follow their recommendation to use a high sampling frequency by setting the sampling interval to 5 seconds.

Building on the pre-averaging concept of Podolskij and Vetter (2009) and Jacod et al. (2009), Christensen et al. (2014) develop a noise-robust realized variance estimator. The pre-averaging of the observed prices in their local neighbourhood of L observations reduces the impact of the noise. The returns based on pre-averaged (log-)prices are given by

$$r^*_{t_i} = \frac{1}{L}\left(\sum_{j=L/2}^{L-1} X_{t_{i+j}} - \sum_{j=0}^{L/2-1} X_{t_{i+j}}\right), \tag{38}$$

where $L = \theta\sqrt{n}$ has to be even. Following Christensen et al.'s (2014) empirical application, we set $\theta = 1$ and employ their noise-robust realized variance estimator

$$\text{RV}_{\text{pa}} = \zeta_L \sum_{i=0}^{N-L+1} |r^*_{t_i}|^2 + \nu_L, \tag{39}$$

where $\zeta_L = \frac{L}{L-K+2}\frac{12}{1+2L^{-2}}$ and $\nu_L = \frac{1}{n-1}\frac{12}{\theta^2(1+2L^{-2})}\sum_{i=2}^{n} r^*_{t_i} r^*_{t_{i-1}}$ serves as a bias-correction.

Finally, we consider the realized bipower variation estimator of Barndorff-Nielsen and Shephard (2006)

$$\text{BPV} = \frac{\pi}{2}\sum_{i=1}^{n-1} |r_{t_{i+1}}||r_{t_i}|. \tag{40}$$

Note that we apply Sheppard's Matlab toolbox '*Oxford Realized*' to compute all return-based measures.[4]

3 Empirical analysis

3.1 Data description and adjustment

We apply the parametric as well as the non-parametric approaches described in Sections 2.5 and 2.6, for the estimation of daily variance for ten actively traded stocks. We investigate the returns and price durations of Apple (AAPL), Bank of America (BAC), The Walt Disney Company (DIS), Evergy Inc (EVRG), Facebook (FB), General Electric (GE), International Business Machines (IBM), Pfizer (PFE), Tesla, Inc. (TSLA) and Walmart (WMT). For each stock, except FB and TSLA, we extract the data for the time period from January 03, 2007 to December 31, 2019, spanning 3272 trading days, from the Trade and Quote (TAQ) database. Owing to later initial public offerings we base our analysis on data beginning on May 18, 2012 for FB and on June 29, 2010 for TSLA, covering 1917 and 2394 trading days, respectively. The TAQ database is inevitably prone to market microstructure and thus requires a certain amount of cleaning. Our data cleaning steps are described in the Appendix. Generally, we

[4] See https://www.kevinsheppard.com/code/matlab/mfe-toolbox/ for further information.

concentrate on trades that occurred between 9:30 and 16:00 and resample the trade data to a second by second frequency, taking volume-weighted price averages within one second.

We define the price duration for each stock as the minimal time required to observe a cumulative change in the price not less than the threshold ι. Since no uniform method for determining the threshold parameter has yet been established in the literature, we follow two approaches to find an appropriate threshold. Firstly, in line with Tse and Yang (2012), we aim to set the value for the daily threshold, such that we obtain an average price duration of 5 minutes. To this end, we define a fine grid of threshold values and choose the threshold that leads to an average duration that comes closest to our target. Secondly, akin to Hong et al. (2023), we define the threshold parameter as the average daily bid/ask-spread multiplied by a factor lying within the range of 3 to 10. In both approaches, we discard all overnight durations following Engle and Russell (1998).

The intensity of trading activities on financial markets changes over the course of a trading day. Typically, the average duration is short at the market open and close, and comparatively longer in between. At the beginning of a day, trading activities are very high due to new events that have occurred during the night (macroeconomic or firm-related news that have become public after the previous market close). At the end of a day, traders tend to close their positions to reduce their exposure to overnight stock market risk.[5] These systematic variations are responsible for the pronounced diurnal seasonality observed in high-frequency financial duration data. However, since these variations are predetermined by market characteristics, we remove the intra-day seasonality component, so that we are able to apply our parametric estimation approaches to the stochastic part of the data. Some studies, like Rodríguez-Poo et al. (2008), directly incorporate the deterministic part in their modelling approach. But as Engle (2000) notes, this procedure does not yield substantial efficiency gains relative to a two-stage adjustment process. Hence, most studies opt for the latter, that is, a prior adjustment of the data. To this end, a variety of adjustment methods have been proposed in the literature (cf. Engle and Russell, 1998; Wu, 2012; Hong et al., 2023; among others, for different adjustment methods). Since Tse and Dong (2014) find that the diurnal adjustment does not have a huge impact on the daily volatility estimation, we follow the

[5] Besides this so-called *time-of-the-day-effect*, early studies based on duration data from the nineties or the start of the new millennium, like inter alia Engle and Russell (1998) and Bauwens et al. (2004), document a distinct *day-of-the-week effect*, which does not seem to be present in our data.

robust and simple-to-implement dummy-adjustment approach of Ghysels et al. (2004)[6]:

We split the 6.5-hour trading time per day into 13 intervals of 30 minutes. Then, we construct an indicator variable

$$y_{ki} = \begin{cases} 1 & \text{if } i \in k, \\ 0 & \text{otherwise} \end{cases} \tag{41}$$

for each 30-minute interval $k = 1, \dots, 13$ and perform a logarithmic dummy variable regression

$$\ln(D_i) = \sum_{k=1}^{13} \alpha_k y_{ki} + u_i, \tag{42}$$

where D_i represents the raw duration data. Finally, we eliminate the *time-of-the-day effect* and obtain the adjusted durations by

$$d_i = D_i \exp\left(-\hat{\alpha}' y_i\right), \tag{43}$$

where $\hat{\alpha} = (\hat{\alpha}_1, \dots, \hat{\alpha}_{13})'$ and $y_i = (y_{1i}, \dots, y_{13i})'$. In line with the aforementioned literature, our durations exhibit a hump-shaped pattern over a trading day. As Figure 1 illustrates, the increase in trading activity at the end of a day is not as distinct as at the beginning of the day.[7] Table 1 reveals that all stocks exhibit overdispersion, right skewness and heavy tails, irrespective of the method used to obtain the raw duration data (5 min. or spread-based). However, the data sets differ in many respects. The mean duration ranges from almost 2 (FB) to more than 12 minutes (PFE) for the spread-based data, whereas for the other data set, the average time for a price event is close to 5 minutes as desired. We also find that the degree of overdispersion, skewness and kurtosis is always substantially higher for the spread-based data. The data adjustment process leads to a small attenuation of the degree of overdispersion, but to an amplification of the other two characteristics for both data sets in most of the cases. Hence, we ascertain that the method used to determine the threshold parameter has a great impact on the received price duration data.

Figure 1 about here

[6] Note that we also apply Nadaraya-Watson kernel regressions and spline functions to remove the diurnal pattern, but we often observe negative durations and sharp declines in the degree of dispersion, and therefore prefer the robust dummy-adjustment approach.

[7] We refrain from displaying the analogous figure for the spread data, since the seasonality pattern is qualitatively similar.

Table 1 about here

3.2 In-sample analysis

In our in-sample analysis, we estimate the daily variance of all ten stocks over the whole sample period using all previously presented estimators. Figure 2 shows the annualized volatility of IBM for an illustrative set of RV estimators ($\text{PDV}^{\text{5min}}_{\text{FHMD, db}}$, $\text{TSRV}^{\text{5sec}}_{\text{db}}$ and RV^{5min}). The trajectories follow each other quite closely, and for all estimates, the volatility peaks during the financial crisis, but, the estimates often differ in their volatility level.

Figure 2 about here

The general tendency, however, that estimates keep close track of each other is also confirmed for the remaining estimates by the correlation heat map in Figure 3, in which we display the correlation coefficients between all realized variance estimates for IBM. In comparison to earlier studies (cf. Patton and Sheppard, 2009), which rely on older high-frequency data (1996-2008), we find a higher overall level of correlation.

Figure 3 about here

We refrain from presenting the results for other stocks, as the outcomes are qualitatively similar.

Our next step is to evaluate the estimation accuracy of all presented RV estimators. In contrast to Tse and Yang (2012) and Hong et al. (2023), who assess the accuracy of their newly introduced price duration-based estimators in the context of a simulation study, we compare their estimation accuracy in a real-world application. We overcome the problem that the object of interest is not observable, not even ex post, by applying the data-based ranking methodology of Patton (2011a). The approach requires an unbiased, but not necessarily precise proxy for the integrated variance. However, since the proxy and the estimators share the same data basis, the estimation errors are likely to be correlated. Therefore, Patton (2011a) recommends employing a one-day lead of the proxy as an instrument to break the correlation.

We measure the estimation accuracy using two well-established loss functions, the root mean squared error (RMSE),

$$\text{RMSE} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(\theta_t - M_{i,t})^2}, \tag{44}$$

and the quasi-likelihood (QLike), that is defined by

$$\text{QLike} = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{\theta_t}{M_{i,t}} - \ln\left(\frac{\theta_t}{M_{i,t}}\right) - 1\right), \tag{45}$$

where θ_t displays the instrument at day t, and $M_{i,t}$ denotes the i-th RV estimator at day t. This setup enables us to obtain a consistent estimate of the difference in accuracy between two competing estimators, so that an iterative testing procedure, such as the model confidence set approach of Hansen et al. (2011) can be applied.

In our in-sample analysis, we estimate the parameters underlying the parametric approaches on a monthly basis, and use the one-day lead of the 5-min RV estimator as an instrument. In Table 2, we report the values for the RMSE and QLike criteria for all stocks, and mark estimators that belong to the 75%-model confidence set. Depending on the considered loss function, the results differ substantially. In general, the sets of surviving models under the RMSE loss function are less sparse than the model confidence sets under the QLike criterion. The final sets are also found to be different in terms of their composition. Thus, for the RMSE loss function, parametric approaches outweigh the model confidence sets, which is evident from the fact that the estimates of daily volatility based on the ACD(-Burr) model using spread data are always part of the model confidence sets for all stocks. Additionally, the FHMD-Burr, as well as the MSMD-Burr model also relying on spread data are included in seven out of ten cases, whereas the most successful return-based approaches ($\text{BPV}_{\text{db}}^{\text{5min}}$ and $\text{BPV}_{\text{ss, db}}^{\text{5min}}$) only appear in three model confidence sets. The good performance of the parametric price duration-based estimators is also reflected in the smallest average losses across all stocks. With 4.58×10^{-4} the ACD-Burr (spread data) has the lowest mean loss across all stocks, closely followed by the FHMD-Burr model (spread data) with an average loss of 4.63×10^{-4}.

However, under the QLike criterion, the composition of the model confidence sets changes. The two-scale realized variance estimator ($\text{TSRV}_{\text{db}}^{\text{5sec}}$) clearly prevails in this situation, since it is part of the surviving set for eight stocks and also exhibits the smallest average loss across all stocks at 0.19. The FHMD-Burr model on spread data appears second most frequently in the model confidence sets and also has the fourth lowest average loss at 0.20. Interestingly, the performance of the two-scale realized variance estimator in terms of the QLike criterion is in stark contrast to its performance under the RMSE loss function, where it is only present in two model confidence sets and has the fourth highest average RMSE-loss across all stocks.

Another noteworthy finding is that the performance of price duration-based estimators is in general better, when they are based on spread data instead of 5-min data.

Table 2 about here

3.3 Out-of-sample analysis

Next, we investigate whether accurate estimates translate into precise forecasts. In order to construct individual variance forecasts for each estimator, we need to employ a time series model that reflects the dynamics of the IV estimates. Owing to the strong persistence of the RV measures, we base our predictions on the heterogeneous autoregressive (HAR) model of Corsi (2009), that accommodates the long-memory feature of realized variance:

$$\theta_{t+h} = \beta_{0,i,h} + \beta_{1,i,h} M_{i,t} + \beta_{2,i,h} \frac{1}{5} \sum_{l=0}^{4} M_{i,t-l} + \beta_{3,i,h} \frac{1}{22} \sum_{l=0}^{21} M_{i,t-l} + u_{i,t}. \tag{46}$$

We estimate the HAR model based on a rolling window of the 500 most recent observations for three forecast horizons, $h = 1, 5, 22$, that represent forecasts one day/week/month ahead. Despite the documented bias in the estimated coefficients of the HAR model in the presence of measurement errors in RV, we do not address this issue directly, unlike Bollerslev et al. (2016) with their HARQ model, but employ a weighted least squares (WLS) estimation technique proposed by Clements and Preve (2021) to overcome this deficiency. In contrast to the usual OLS, WLS places less weight on observations, for which the errors are likely to be large, leading to more efficient estimators. Clements and Preve (2021) provide evidence by means of a large-scale out-of-sample forecasting study that the HAR model estimated with WLS exhibits higher predictive accuracy than current state-of-the-art models or extensions of the original model, like e.g. the HARQ model (Bollerslev et al., 2016) or the Leverage HAR model (Corsi and Reno, 2012). Again, we employ the 5-min RV estimates as the variance proxy and use the RMSE and QLike criteria to assess the forecast performance following Patton (2011b). To rule out any look-ahead bias, we estimate the duration-based models day-by-day using the 21 most recent days of intra-day observations.

We display the relative forecasting performance compared to the 5-min RV estimator for each stock in Tables 5 - 14. Analogously to the in-sample analysis, we mark the models belonging to the 75% model confidence sets. In Table 3, we summarize the results and report how often a particular model is part of the final surviving set. Across all stocks and for each forecasting horizon, the parametric duration estimator $\text{PDV}^{\text{Spread}}_{\text{ACD, db}}$ exhibits the highest precision in terms of the RMSE and QLike criteria in most cases (cf. Tables 5 - 14). Especially in terms of the QLike criterion, we find large improvements up to 12% compared to the benchmark model $\text{RV}^{5\text{min}}$ (cf. Table 14). This is also reflected in the highest number of inclusions in the model confidence sets. In 58 out of 60 cases, the $\text{PDV}^{\text{Spread}}_{\text{ACD, db}}$ belongs to the final set. The best return-based method, the $\text{BPV}^{5\text{min}}_{\text{ss, db}}$, follows at a wide gap with only 30 inclusions. The other duration-based approaches also do not appear frequently in the model confidence set, e.g. the $\text{PDV}^{\text{Spread}}_{\text{FHMD-Burr,db}}$ has a total of 30 inclusions. In general, we find that the forecasting performance of the parametric price duration-based approaches hinges on the specification underlying the PDV. Moreover, the accuracy of all parametric duration methods is once again higher when they are based on spread-data instead of 5-min data.

Table 3 about here

3.4 Value-at-Risk forecasting

Since the introduction by the first pillar of the Basel 2 Accord of the Value-at-Risk (VaR) as the leading market risk measure, it has become pivotal for any bank's minimum capital requirements. Due to its importance, several different methods have been proposed to forecast VaR as accurately as possible, see Nieto and Ruiz (2016) for a recent survey. The vast majority of studies relies on daily return data to forecast the VaR. One of the most popular approaches imputes that the conditional distribution of returns can be described by

$$r_t = \mu_t + \epsilon_t, \quad \epsilon_t = \sigma_t u_t, \text{ where } u_t \overset{iid}{\sim} (0, 1). \tag{47}$$

Based on this location-scale family setting, the resulting one-step-ahead $100\alpha\%$ VaR, conditional on the information set available at time $t-1$, can be computed by

$$\text{VaR}^{\alpha}_{t|t-1} = \mu_t + \sigma_t F^{-1}(\alpha), \tag{48}$$

where $F^{-1}(\alpha)$ displays the α-quantile of the distribution of u_t. As a major benefit, this framework offers a wide spectrum of models and distributions that can be employed to represent the dynamics of the conditional mean, μ_t, the conditional variance, σ_t^2, and the distribution of the innovation, u_t. The simple GARCH(1,1) model (Bollerslev, 1986), coupled with the assumption of a constant conditional mean return, is a common choice among both practitioners and academics for modelling the conditional variance. Moreover, the Student-t distribution is often adopted in order to reflect the leptokurtotic nature of the returns. Expressing this modeling approach in the framework presented in Eq. (47), we have

$$\begin{gathered} r_t = \mu + \epsilon_t, \quad \epsilon_t = \sigma_t u_t, \text{ where } u_t \overset{iid}{\sim} t_\nu(0,1), \\ \sigma_t^2 = \beta_0 + \beta_1 \sigma_{t-1}^2 + \beta_2 \epsilon_{t-1}^2. \end{gathered} \tag{49}$$

However, some recent approaches employ intra-day data in the form of different RV measures for VaR forecasting with promising results, like *inter alia* Giot and Laurent (2004), Clements et al. (2008) or Fuertes and Olmo (2013). Given the success of the parametric price duration-based approaches in the in-sample as well as in the out-of-sample analysis, we examine whether the usage of RV measures based on high-frequency duration data can offer benefits compared to daily and intra-day return data. For incorporating RV measures in the VaR predictions, we need to have (i) a law of motion for the RV dynamics and (ii) a mapping between the conditional expectation of the RV measure and the conditional variance of the returns (cf. Brownless and Gallo, 2010). Most approaches rely on a two-step procedure that builds, for example, on a HAR or ARFIMA(X) model (cf. Giot and Laurent, 2004; Clements et al., 2008) in the first instance and relates the conditional RV prediction with the conditional variance by means of a linear function, which serves as a bias-correcting mechanism, in a consecutive step. A more novel approach from Maheu and McCurdy (2011) combines the two steps into one by using a bivariate model. Owing to its success in the closely related field of return density forecasting, we follow their approach and model the return and the RV simultaneously in the following way[8]

$$r_t = \mu + \epsilon_t, \quad \epsilon_t = \sigma_t u_t, \text{ where } u_t \sim t_\nu(0,1),$$
$$\log(\mathrm{RV}_t) = \omega + \phi_1 \log(\mathrm{RV}_{t-1}) + \phi_2 \log(\mathrm{RV}_{t-5,5}) + \phi_3 \log(\mathrm{RV}_{t-22,22}) + v_t, \tag{50}$$

where $\mathrm{RV}_{t-j,j} = \frac{1}{j}\sum_{i=1}^{j} \mathrm{RV}_{t-i}$ and $v_t \sim \mathcal{N}(0,1)$. The authors argue that - under certain empirically realistic conditions - the conditional expectation of an unbiased RV measure is equal to the conditional variance of returns, i.e. $\mathbb{E}_{t-1}(\mathrm{RV}_t) = \mathbb{V}\mathrm{ar}_{t-1}(r_t) = \sigma_t^2$. Accordingly, they establish a linkage between these two measures. Due to the log-normal distribution of the RV measure in this specification, the connection is given by

$$\sigma_t^2 = \mathbb{E}_{t-1}(RV_t) = \exp\Big(\mathbb{E}_{t-1}\left(\log(\mathrm{RV}_t)\right) + 0.5\mathbb{V}\mathrm{ar}_{t-1}\left(\log(\mathrm{RV}_t)\right)\Big). \tag{51}$$

Based on this framework, we employ duration-based as well as return-based RV measures for our VaR forecasting exercise. In our analysis, we concentrate on the most successful RV measures from Sections 3.2 and 3.3. Relying on the $\mathrm{PDV}^{\mathrm{Spread}}_{\mathrm{FHMD\text{-}Burr,db}}$ and the $\mathrm{PDV}^{\mathrm{Spread}}_{\mathrm{ACD,\,db}}$ as duration-based RV measures and on the $\mathrm{RV}^{5\mathrm{min}}$, the $\mathrm{TSRV}^{5\mathrm{sec}}_{\mathrm{db}}$ as well as the $\mathrm{BPV}^{5\mathrm{min}}_{\mathrm{ss,\,db}}$ as return-based benchmarks, we estimate the bivariate model via maximum-likelihood and project the conditional return variance one-day ahead to obtain a VaR forecast. Doing so for a rolling window of 500 observations, we obtain a series of 2752 VaR predictions for the time span December 2008 until December 2019.[9]

A suitable illustration of our VaR predictions using the $\mathrm{RV}^{5\mathrm{min}}$ and $\mathrm{PDV}^{\mathrm{Spread}}_{\mathrm{ACD,db}}$ measures as components for the bivariate model for IBM against its returns in Figure 4, reveals that our forecasts sometimes fail to quantify the maximum loss that will not be exceeded with 95% probability and that the returns occasionally exceed (or 'hit') the threshold set by the forecasted VaR. As long as the number of failures ('hits') is within the range of expectable exceedances, the forecasting approach can be regarded as adequate. In Figure 4, we see that for both approaches, the VaR violations often coincide. Furthermore, the total numbers of 'hits' (137 and 140 for $\mathrm{RV}^{5\mathrm{min}}$ and $\mathrm{PDV}^{\mathrm{Spread}}_{\mathrm{FHMD\text{-}Burr,db}}$, respectively) are close to the expected number of 137.55.

[8] Other options for characterising the dynamics of the log-RV than the HAR model presented above are also viable and are described by Maheu and McCurdy (2011).

[9] For FB and TSLA, the out-of-sample period is reduced.

Figure 4 about here

However, the number of 'hits' is a very uninformative indicator and does not reflect the trade-off between the costs of over- and under-prediction (regulatory penalty vs. opportunity cost of capital). In order to account for these conflicting targets, we compare the forecasting accuracy on the basis of the asymmetric tick-loss function of Giacomini and Komunjer (2005), which is given by

$$\mathrm{TL}_\alpha\left(r_{t+1}, \mathrm{VaR}_{t+1|t}\right) = \left(\alpha - \mathbb{1}_{\{r_{t+1} < \mathrm{VaR}^\alpha_{t+1|t}\}}\right)\left(r_{t+1} - \mathrm{VaR}^\alpha_{t+1|t}\right). \tag{52}$$

For 5%- as well as 1%-VaR forecasts, we present the average tick-loss function values for each stock in Table 4. For each row, we highlight the best forecast in bold numbers. To test for the predictive power of the duration-based RV measure stemming from the ACD model, we conduct pairwise comparisons via the Diebold-Mariano test (1995) in the framework of Giacomini and White (2006).[10] We display significant outperformance (underperformance) by *** (†††), ** (††), and * (†) for the 1%, 5% and 10% significance levels. Apart from our intra-day based RV measures, we opt for the classic location-scale family approach in combination with a GARCH(1,1) model as described in Eq. (49) as a benchmark.

Table 4 about here

According to Table 4, the predictions of future VaR values based on the $\mathrm{PDV}^{\mathrm{Spread}}_{\mathrm{ACD,\,db}}$ measure perform best in terms of the tick-loss function for almost every stock and confidence level. In comparison to other forecasts, we find that the $\mathrm{PDV}^{\mathrm{Spread}}_{\mathrm{ACD,\,db}}$-based forecasting approach is significantly better than the other forecasting methods in the vast majority of cases. In particular, compared to GARCH model-based forecasts, we find highly significant outperformance in favour of the duration-based approach. This result also holds for all other VaR forecasts that rely on high-frequency intra-day data. They furthermore, perform substantially better than predictions that only employ daily return data (GARCH).

Among all competitors, VaR forecasts utilizing the estimates of the other duration-based

[10] We note that the asymmetric tick-loss function defined in Eq. (52) is not differentiable due to the presence of the indicator function. The non-differentiability may cause a problem in the implementation of the Diebold-Mariano test. However, as pointed out by Granger (1999), the issue is just a technicality, due to the fact that it is always possible to find a smooth function which can approximate the non-differentiable one arbitrarily closely.

variance measure, $\text{PDV}^{\text{Spread}}_{\text{FHMD-Burr,db}}$, seem to be a viable option. They often perform slightly better than return-based methods. However, in comparison to predictions on the basis of $\text{PDV}^{\text{Spread}}_{\text{ACD,db}}$ estimates, we often find significantly inferior forecasting accuracy.

4 Conclusion

As a result of the availability of high-frequency asset price data, numerous ways to estimate the asset price variability have been proposed. These estimators, known as RV measures, are all based on evenly-spaced intra-day return data. A more recent approach allows the estimation of the asset's integrated variance by employing irregularly-spaced price duration data. The usage of intra-day duration data is more robust than return data and ensures that the information content of the data can be extracted more efficiently. However, an integral component of the newly-introduced estimator is the specification of a financial duration model. Until now, models from the ACD family have been the standard choice. However, motivated by the success of alternative duration models, we propose new duration-based realized variance estimators based on the FIACD and the Log-ACD, as well as two hidden Markov duration models, the MSMD and the FHMD model. For both of them, we derive the price duration-based estimator.

In an in-sample analysis, we apply the duration-based estimators and a set of well-established return-based RV measures to estimate the daily integrated variance of ten highly liquid stocks. We compare the estimation accuracy on the basis of intra-day data from January 2007 to December 2019 and find that (i) the duration-based approaches, in particular the ACD- and FHMD-based estimates, exhibit high estimation precision under the RMSE criterion and (ii) the TSRV measure and the FHMD-based estimator perform best in terms of the QLike loss. Moreover, we evaluate our duration-based variance estimators in two forecasting exercises. First, we construct predictions of future variance on the basis of a HAR model. Using the model confidence approach, we find that the ACD-based forecasts are almost always part of the final surviving set, irrespective of the loss function and forecast horizon. Focusing on the best-performing duration- and return-based estimators in our second forecasting application, we employ our daily estimates of the realized variance to predict the VaR one-day-ahead. On the basis of a Diebold-Mariano test, we find that the ACD-based method often provides significant accuracy gains. Our results indicate that the informational content in the irregularly-spaced price duration data can help in improving the estimation and forecasting accuracy, especially by employing the ACD(1,1) as the underlying duration model.

In this paper, we separately investigate estimators for the integrated variance for ten different stocks. However, employing high-frequency data to produce accurate estimates of the covariance matrix might also be highly relevant for portfolio optimisation and risk management. In the domain of intra-day returns, multivariate analogons to popular RV measures have been already proposed, such as, *inter alia*, the two-scale integrated covariance estimator of Zhang (2011) and the multivariate realized kernel estimator of Barndorff-Nielsen et al. (2011). Extending the parametric duration-based approach to the multivariate setting may therefore be a promising path for future research.

Appendix: A. Data cleaning

We retain only trades with positive trade prices and volumes, and valid trade correction indicators "00" and "01". Following Aït-Sahalia et al. (2020), we further exclude trade sale conditions, "Z", "B", "U", "T", "L", "G", "W", "K", and "J", as well as an accompanying "I" for odd lot trades. We take the median price if multiple trades are recorded at the same timestamp, which further reduces recording errors. This procedure however, does not yet guarantee fully reliable data. When merging the trade and quote observations within one microsecond, we apply the following two filters: (i) we remove all crossed quotes, i.e. all quotes for which a higher bid than ask-price is reported and (ii) we exclude all transaction prices which are smaller than the bid-price minus the spread, or resp. greater than the ask-price plus the spread.

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Tables and Figures

Table 1: Descriptive statistics of the raw and adjusted duration data

		Raw duration data					Adjusted duration data			
		No. of obs.	Mean	Overdisp.	Skewness	Kurtosis	Mean	Overdisp.	Skewness	Kurtosis
5 min. data	AAPL	251,348	299.731	1.346	3.768	30.547	1.613	1.072	3.416	31.136
	BAC	253,726	296.695	1.406	4.057	33.295	1.689	1.142	3.718	38.271
	DIS	251,566	299.364	1.359	3.509	24.091	1.714	1.175	3.823	31.864
	EVRG	251,864	299.244	1.419	4.161	36.818	1.858	1.210	4.210	50.773
	FB	146,714	299.486	1.378	3.787	30.656	1.597	1.059	3.372	33.972
	GE	253,982	297.236	1.412	4.013	33.176	1.755	1.180	3.556	28.246
	IBM	251,565	299.895	1.385	3.814	30.253	1.722	1.156	3.578	28.198
	PFE	253,489	298.128	1.368	3.688	26.967	1.744	1.159	3.321	23.325
	TSLA	182,464	299.669	1.534	4.363	37.765	1.800	1.296	5.779	94.962
	WMT	251,848	299.485	1.412	3.906	32.217	1.716	1.164	3.704	34.967
Spread data	AAPL	168,165	430.600	1.957	6.549	71.632	1.709	1.309	8.047	206.391
	BAC	585,670	127.545	2.771	10.489	209.479	1.746	1.394	13.949	1151.886
	DIS	253,402	294.969	1.668	5.455	61.693	1.797	1.338	7.845	233.522
	EVRG	181,249	409.463	1.748	5.554	55.344	1.862	1.357	7.469	179.940
	FB	396,104	111.695	1.841	9.759	259.408	1.699	1.244	5.898	118.267
	GE	378,093	198.547	2.114	8.096	138.145	1.828	1.372	5.816	97.187
	IBM	252,155	296.280	1.727	5.569	60.275	1.777	1.233	4.153	39.834
	PFE	95,662	747.073	1.721	4.597	35.803	1.783	1.335	8.736	266.290
	TSLA	261,735	207.892	1.990	8.367	147.362	1.858	1.415	8.248	220.006
	WMT	128,303	526.859	1.796	4.946	42.316	1.823	1.616	19.660	910.529

Note: For each stock we report the descriptive statistics for the (raw and adjusted) 5 min. data in the top panel and for the (raw and adjusted) spread-based data in the bottom panel.

Table 2: In-sample performance of realized measures (Proxy: 5-min RV)

	AAPL		BAC		DIS		EVRG		FB	
	RMSE	QLIKE	RMSE	QLIKE	RMSE	QLIKE	RMSE	QLIKE	RMSE	QLIKE
RV^{5min}	5.064	0.279	14.013	0.225	2.900	0.211	4.292	0.237	3.787	0.231
RV^{5min}_{ss}	5.317	0.279	15.106	0.219	3.487	0.208	4.263	0.220	3.461	0.229
RK^{5sec}_{parzen}	5.296	0.265	15.750	0.224	3.860	0.201	4.261	0.223	3.424	0.218
RK^{5sec}_{cubic}	5.479	0.258	16.104	0.219	3.842	0.193	4.185	0.211	3.386	0.213
RK^{5sec}_{th2}	6.211	0.245	15.701	0.197	3.767	0.183	4.332	0.193	3.388	0.204
$RK^{5sec}_{bartlett}$	5.899	0.245	15.634	0.197	3.741	0.182	4.310	0.192	3.391	0.203
$TSRV^{5sec}_{db}$	5.061	0.230	15.181	0.181	3.357	0.171	5.417	0.189	3.808	0.190
$MSRV^{5sec}$	4.578	0.675	14.071	0.417	3.101	0.510	4.221	0.235	3.490	0.601
RV^{5min}_{pa}	4.754	0.627	14.791	0.538	3.017	0.464	4.794	0.684	3.529	0.506
BPV^{5min}_{db}	5.248	0.313	13.100	0.259	2.689	0.234	4.260	0.280	3.431	0.258
$BPV^{5min}_{ss,db}$	5.281	0.309	14.796	0.242	3.187	0.229	4.057	0.258	3.297	0.253
$PDV^{5min}_{FHMD,db}$	4.366	0.284	14.134	0.206	2.965	0.231	3.918	0.388	3.271	0.242
$PDV^{5min}_{FHMD\text{-}Burr,db}$	4.380	0.282	13.951	0.204	2.987	0.228	3.915	0.385	3.141	0.240
$PDV^{Spread}_{FHMD,db}$	4.196	0.241	12.893	0.184	2.605	0.185	3.726	0.262	2.993	0.228
$PDV^{Spread}_{FHMD\text{-}Burr,db}$	4.148	0.234	12.853	0.181	2.608	0.182	3.757	0.257	2.981	0.222
$PDV^{5min}_{MSMD,db}$	4.494	0.283	13.969	0.207	3.108	0.229	3.920	0.390	3.199	0.241
$PDV^{5min}_{MSMD\text{-}Burr,db}$	4.464	0.279	14.176	0.203	3.009	0.227	3.916	0.386	3.220	0.237
$PDV^{Spread}_{MSMD,db}$	4.216	0.250	12.893	0.189	2.616	0.191	3.748	0.272	3.004	0.236
$PDV^{Spread}_{MSMD\text{-}Burr,db}$	4.203	0.241	13.075	0.187	2.630	0.187	3.801	0.264	2.987	0.230
$PDV^{5min}_{ACD,\ db}$	4.377	0.268	13.603	0.189	3.049	0.224	4.033	0.386	3.047	0.235
$PDV^{5min}_{ACD\text{-}Burr,db}$	4.371	0.269	13.473	0.190	2.961	0.223	4.007	0.387	3.089	0.235
$PDV^{Spread}_{ACD,db}$	4.086	0.236	12.871	0.182	2.561	0.181	3.705	0.261	2.979	0.229
$PDV^{Spread}_{ACD\text{-}Burr,db}$	4.088	0.237	12.762	0.182	2.561	0.185	3.706	0.261	2.982	0.229
$PDV^{5min}_{FIACD,db}$	4.546	1.004	14.572	0.815	3.074	0.818	3.829	1.205	3.527	0.846
$PDV^{5min}_{FIACD\text{-}Burr,db}$	4.880	1.081	14.365	0.795	3.140	0.841	3.775	1.391	3.554	0.911
$PDV^{Spread}_{FIACD,db}$	4.461	12.446	13.023	3.449	2.905	2.304	3.893	5.646	3.107	0.388
$PDV^{Spread}_{FIACD\text{-}Burr,db}$	4.463	11.267	13.004	3.305	2.875	2.021	3.880	5.177	3.111	0.386
$PDV^{5min}_{Log\text{-}ACD,db}$	4.383	0.268	13.576	0.189	2.993	0.223	4.038	0.384	3.052	0.234
$PDV^{5min}_{Log\text{-}ACD\text{-}Burr,db}$	4.376	0.269	13.455	0.191	2.929	0.223	4.015	0.385	3.063	0.234
$PDV^{Spread}_{Log\text{-}ACD,db}$	4.089	0.239	12.831	0.181	2.574	0.182	3.705	0.258	2.983	0.227
$PDV^{Spread}_{Log\text{-}ACD\text{-}Burr,db}$	4.076	0.244	12.673	0.183	2.562	0.184	3.701	0.258	2.992	0.231
$NPDV^{5min}_{db}$	4.405	0.272	14.039	0.192	3.050	0.225	3.986	0.402	3.095	0.233
$NPDV^{Spread}_{db}$	4.298	0.278	13.150	0.203	2.656	0.212	3.771	0.301	3.028	0.253

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Table 2: Continued.

	GE		IBM		PFE		TSLA		WMT	
	RMSE	QLIKE	RMSE	QLIKE	RMSE	QLIKE	RMSE	QLIKE	RMSE	QLIKE
RV^{5min}	6.168	0.207	2.470	0.199	2.060	0.186	9.221	0.273	2.303	0.244
RV^{5min}_{ss}	6.204	0.207	2.442	0.198	2.204	0.181	8.846	0.268	2.856	0.239
RK^{5sec}_{parzen}	6.457	0.207	2.432	0.191	2.222	0.179	8.929	0.264	2.696	0.230
RK^{5sec}_{cubic}	6.596	0.203	2.371	0.181	2.337	0.172	9.034	0.250	2.975	0.217
RK^{5sec}_{th2}	6.468	0.185	2.574	0.170	2.461	0.158	8.817	0.229	2.926	0.206
$RK^{5sec}_{bartlett}$	6.428	0.185	2.527	0.169	2.420	0.157	8.868	0.229	2.879	0.205
$TSRV^{5sec}_{db}$	6.095	0.174	2.809	0.160	2.251	0.149	9.086	0.209	2.314	0.198
$MSRV^{5sec}$	5.907	0.387	2.418	0.465	2.121	0.362	9.489	0.412	2.187	0.589
RV^{5min}_{pa}	5.864	0.557	2.357	0.497	1.985	0.514	10.156	0.701	2.507	0.618
BPV^{5min}_{db}	5.981	0.243	2.538	0.223	1.986	0.214	8.739	0.311	2.305	0.268
$BPV^{5min}_{ss,db}$	6.005	0.234	2.448	0.217	2.166	0.207	8.470	0.300	2.740	0.266
$PDV^{5min}_{FHMD,db}$	5.647	0.203	2.333	0.209	2.021	0.196	8.578	0.298	2.241	0.269
$PDV^{5min}_{FHMD\text{-}Burr,db}$	5.641	0.201	2.332	0.207	2.022	0.193	8.698	0.297	2.236	0.266
$PDV^{Spread}_{FHMD,db}$	5.280	0.153	2.180	0.176	1.778	0.142	8.682	0.246	1.994	0.193
$PDV^{Spread}_{FHMD\text{-}Burr,db}$	5.263	0.151	2.224	0.173	1.781	0.141	8.662	0.241	1.990	0.191
$PDV^{5min}_{MSMD,db}$	5.795	0.203	2.374	0.208	2.142	0.195	8.584	0.299	2.372	0.266
$PDV^{5min}_{MSMD\text{-}Burr,db}$	5.720	0.199	2.418	0.205	2.038	0.192	8.584	0.294	2.273	0.266
$PDV^{Spread}_{MSMD,db}$	5.318	0.160	2.207	0.185	1.825	0.150	8.705	0.255	2.013	0.200
$PDV^{Spread}_{MSMD\text{-}Burr,db}$	5.305	0.157	2.225	0.178	1.815	0.147	8.777	0.251	2.013	0.193
$PDV^{5min}_{ACD,db}$	5.618	0.180	2.384	0.203	2.088	0.174	8.544	0.297	2.254	0.257
$PDV^{5min}_{ACD\text{-}Burr,db}$	5.548	0.181	2.387	0.204	2.085	0.177	8.433	0.297	2.269	0.257
$PDV^{Spread}_{ACD,db}$	5.255	0.154	2.106	0.174	1.734	0.147	8.690	0.250	1.970	0.185
$PDV^{Spread}_{ACD\text{-}Burr,db}$	5.248	0.154	2.126	0.178	1.746	0.144	8.655	0.250	1.973	0.187
$PDV^{5min}_{FIACD,db}$	6.219	0.527	2.399	0.737	2.166	0.558	8.845	0.677	2.275	0.623
$PDV^{5min}_{FIACD\text{-}Burr,db}$	6.053	0.619	2.448	0.782	2.084	0.647	10.979	0.860	2.154	0.741
$PDV^{Spread}_{FIACD,db}$	5.570	1.748	2.249	3.157	2.445	14.044	9.795	1.578	2.155	8.194
$PDV^{Spread}_{FIACD\text{-}Burr,db}$	5.575	1.660	2.233	2.750	2.430	13.399	9.713	1.211	2.138	7.711
$PDV^{5min}_{Log\text{-}ACD,db}$	5.611	0.180	2.385	0.204	2.084	0.175	8.555	0.297	2.261	0.256
$PDV^{5min}_{Log\text{-}ACD\text{-}Burr,db}$	5.543	0.181	2.385	0.204	2.080	0.177	8.447	0.298	2.274	0.256
$PDV^{Spread}_{Log\text{-}ACD,db}$	5.269	0.154	2.107	0.175	1.764	0.149	8.699	0.245	1.974	0.189
$PDV^{Spread}_{Log\text{-}ACD\text{-}Burr,db}$	5.269	0.155	2.120	0.179	1.755	0.149	8.676	0.247	1.970	0.185
$NPDV^{5min}_{db}$	5.587	0.182	2.462	0.206	2.077	0.180	8.369	0.298	2.262	0.259
$NPDV^{Spread}_{db}$	5.354	0.170	2.242	0.196	1.876	0.188	8.768	0.281	2.036	0.237

Note: We report the average RMSE and QLike losses. The RMSE means are multiplied by factor 10,000 for the sake of legibility. For some estimators, we perform a small-sample adjustment, which we abbreviate as 'db' in the subscript. The sampling frequency (e.g. 5min) of each RV estimator is attached to its superscript. A gray-shaded cell indicates that the associated estimate belongs to the 75% model confidence set.

Table 3: Model confidence set inclusion frequency

	1 day ahead		1 week ahead		1 month ahead		Total		
	RMSE	QLIKE	RMSE	QLIKE	RMSE	QLIKE	RMSE	QLIKE	Σ
RV^{5min}	3	0	1	2	10	10	14	12	26
RV^{5min}_{ss}	4	2	1	2	10	9	15	13	28
RK^{5sec}_{parzen}	4	3	2	2	10	9	16	14	30
RK^{5sec}_{cubic}	4	2	1	2	10	9	15	13	28
RK^{5sec}_{th2}	4	1	1	2	10	9	15	12	27
$RK^{5sec}_{bartlett}$	4	1	1	2	10	9	15	12	27
$TSRV^{5sec}_{db}$	3	0	1	2	10	9	14	11	25
$MSRV^{5sec}$	3	0	1	1	9	8	13	9	22
RV^{5min}_{pa}	4	0	3	3	10	9	17	12	29
BPV^{5min}_{db}	3	1	3	2	10	9	16	12	28
$BPV^{5min}_{ss,db}$	4	2	2	3	10	9	16	14	30
$PDV^{5min}_{FHMD,db}$	3	1	1	2	10	9	14	12	26
$PDV^{5min}_{FHMD\text{-}Burr,db}$	4	1	2	2	10	9	16	12	28
$PDV^{Spread}_{FHMD,db}$	3	0	1	2	10	10	14	12	26
$PDV^{Spread}_{FHMD\text{-}Burr,db}$	4	0	3	3	10	10	17	13	30
$PDV^{5min}_{MSMD,db}$	3	1	1	2	10	9	14	12	26
$PDV^{5min}_{MSMD\text{-}Burr,db}$	4	1	1	2	10	9	15	12	27
$PDV^{Spread}_{MSMD,db}$	3	0	1	2	10	8	14	10	24
$PDV^{Spread}_{MSMD\text{-}Burr,db}$	4	0	1	2	10	10	15	12	27
$PDV^{5min}_{ACD,db}$	5	1	1	1	10	10	16	12	28
$PDV^{5min}_{ACD\text{-}Burr,db}$	5	1	1	2	10	10	16	13	29
$PDV^{Spread}_{ACD,db}$	10	10	10	8	10	10	30	28	58
$PDV^{Spread}_{ACD\text{-}Burr,db}$	6	5	5	3	10	10	21	18	39
$PDV^{5min}_{FIACD,db}$	2	0	1	1	10	9	13	10	23
$PDV^{5min}_{FIACD\text{-}Burr,db}$	2	0	1	1	10	9	13	10	23
$PDV^{Spread}_{FIACD,db}$	4	1	3	5	10	9	17	15	32
$PDV^{Spread}_{FIACD\text{-}Burr,db}$	4	1	3	5	10	9	17	15	32
$PDV^{5min}_{Log\text{-}ACD,db}$	5	1	1	1	10	10	16	12	28
$PDV^{5min}_{Log\text{-}ACD\text{-}Burr,db}$	4	1	1	1	10	10	15	12	27
$PDV^{Spread}_{Log\text{-}ACD,db}$	9	6	7	7	10	10	26	23	49
$PDV^{Spread}_{Log\text{-}ACD\text{-}Burr,db}$	6	4	7	4	10	10	23	18	41
$NPDV^{5min}_{db}$	4	1	2	2	10	10	15	13	28
$NPDV^{Spread}_{db}$	3	0	1	1	10	9	14	10	24

Note: For each RV estimator we report the number of inclusions in the 75% model confidence set across all ten stocks for both loss functions (RMSE and QLike) and for three forecast horizons (1 day-, 1 week- and 1 month-ahead).

Table 4: Out-of-sample VaR one-day ahead forecasts evaluated by tick loss function

		GARCH	RV^{5min}	$TSRV^{5sec}_{db}$	$BPV^{5min}_{ss,\,db}$	$PDV^{Spread}_{FHMD\text{-}Burr,db}$	$PDV^{Spread}_{ACD,\,db}$
AAPL	5%	0.1956***	0.1820**	0.1815**	0.1817**	0.1813***	**0.1801**
	1%	0.0642***	0.0569	0.0569	0.0569	0.0568	**0.0567**
BAC	5%	0.3110***	0.2546	**0.2514**	0.2539	0.2565***	0.2544
	1%	0.1123***	0.0786	0.0767	0.0781	0.0768**	**0.0756**
DIS	5%	0.1646***	0.1502**	0.1494***	0.1484	0.1482**	**0.1473**
	1%	0.0580***	0.0459***	0.0454***	0.0449**	0.0440	**0.0438**
EVRG	5%	0.1259***	0.1212	0.1212**	0.1207	0.1205***	**0.1197**
	1%	0.0428***	0.0382	0.0386**	0.0379	0.0379**	**0.0374**
FB	5%	0.2147***	0.2027***	0.2015**	0.2020**	0.2001	**0.1997**
	1%	0.0796**	0.0714	0.0718**	0.0713	0.0706	**0.0705**
GE	5%	0.2016***	0.1811	**0.1772**	0.1802	0.1794	0.1792
	1%	0.0631***	0.0529	**0.0517**††	0.0523	0.0534	0.0532
IBM	5%	0.1576***	0.1431	0.1429**	0.1424	0.1422*	**0.1416**
	1%	0.0568***	0.0515	0.0517	0.0517	0.0517***	**0.0510**
PFE	5%	0.1353***	0.1288***	0.1273	0.1286***	0.1276**	**0.1261**
	1%	0.0404***	0.0366**	0.0353	0.0363**	0.0362***	**0.0351**
TSLA	5%	0.3525	**0.3507**	0.3534**	0.3507	0.3516	0.3508
	1%	0.1185	0.1148	0.1158	0.1145	0.1143	**0.1142**
WMT	5%	0.1281***	0.1230**	0.1226**	0.1226**	0.1225***	**0.1212**
	1%	0.0460***	0.0416**	0.0411	0.0408	0.0409***	**0.0400**

Note: We report the average tick loss function values for 5% and 1% -VaR one-day-ahead forecast. Bold numbers denote the lowest average tick loss values for each considered stock. We test for significant improvements (deteriorations) via the Diebold-Mariano test and report the significance levels 1%, 5% and 10% by *** (†††), ** (††), and * (†).

Table 5: Relative forecasting performance for AAPL

	1 day ahead		1 week ahead		1 month ahead	
	RMSE	QLIKE	RMSE	QLIKE	RMSE	QLIKE
RV^{5min}	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
RV^{5min}_{ss}	1.0455	0.9818	1.0058	0.9948	1.0030	0.9892
RK^{5sec}_{parzen}	1.0444	0.9809	1.0055	0.9926	1.0033	0.9908
RK^{5sec}_{cubic}	1.0453	0.9841	1.0056	0.9910	1.0036	0.9879
RK^{5sec}_{th2}	1.0473	0.9837	1.0069	0.9909	1.0050	0.9834
$RK^{5sec}_{bartlett}$	1.0403	0.9817	1.0060	0.9903	1.0041	0.9849
$TSRV^{5sec}_{db}$	1.0050	0.9904	1.0005	0.9991	1.0007	0.9950
$MSRV^{5sec}$	1.0050	0.9917	1.0005	0.9993	1.0008	0.9956
RV^{5min}_{pa}	0.9986	0.9983	1.0013	1.0082	0.9993	1.0010
BPV^{5min}_{db}	1.0019	1.0066	1.0007	0.9995	0.9989	0.9953
$BPV^{5min}_{ss,\ db}$	1.0442	0.9836	1.0067	0.9963	1.0023	0.9850
$PDV^{5min}_{FHMD,\ db}$	1.0025	0.9740	1.0000	0.9984	0.9997	0.9922
$PDV^{5min}_{FHMD\text{-}Burr,db}$	0.9997	0.9734	0.9994	0.9986	0.9994	0.9918
$PDV^{Spread}_{FHMD,\ db}$	1.0131	0.9602	0.9998	0.9844	1.0020	1.0002
$PDV^{Spread}_{FHMD\text{-}Burr,db}$	1.0129	0.9615	0.9997	0.9870	1.0020	1.0014
$PDV^{5min}_{MSMD,\ db}$	1.0026	0.9749	1.0001	0.9991	0.9997	0.9919
$PDV^{5min}_{MSMD\text{-}Burr,db}$	1.0108	0.9825	1.0020	1.0034	1.0010	0.9945
$PDV^{Spread}_{MSMD,\ db}$	1.0057	0.9600	0.9997	0.9907	1.0015	1.0067
$PDV^{Spread}_{MSMD\text{-}Burr,db}$	1.0056	0.9569	0.9986	0.9868	1.0008	1.0021
$PDV^{5min}_{ACD,\ db}$	0.9877	0.9709	0.9955	0.9920	0.9984	0.9959
$PDV^{5min}_{ACD\text{-}Burr,db}$	0.9878	0.9725	0.9954	0.9920	0.9984	0.9962
$PDV^{Spread}_{ACD,\ db}$	0.9874	0.9363	0.9942	0.9762	0.9998	1.0043
$PDV^{Spread}_{ACD\text{-}Burr,db}$	0.9943	0.9330	0.9957	0.9802	1.0006	1.0031
$PDV^{5min}_{FIACD,\ db}$	1.0087	1.1203	0.9996	1.0028	0.9976	0.9779
$PDV^{5min}_{FIACD\text{-}Burr,db}$	1.0099	1.1545	0.9985	1.0053	0.9976	0.9764
$PDV^{Spread}_{FIACD,\ db}$	1.0293	1.1998	1.0041	1.0063	0.9931	0.9303
$PDV^{Spread}_{FIACD\text{-}Burr,db}$	1.0281	1.2160	1.0023	1.0027	0.9933	0.9542
$PDV^{5min}_{Log\text{-}ACD,db}$	0.9882	0.9724	0.9956	0.9925	0.9984	0.9960
$PDV^{5min}_{Log\text{-}ACD\text{-}Burr,db}$	0.9883	0.9741	0.9956	0.9924	0.9984	0.9964
$PDV^{Spread}_{Log\text{-}ACD,db}$	0.9848	0.9412	0.9946	0.9809	0.9995	1.0034
$PDV^{Spread}_{Log\text{-}ACD\text{-}Burr,db}$	0.9890	0.9409	0.9950	0.9814	1.0002	1.0040
$NPDV^{5min}_{db}$	0.9903	0.9798	0.9962	0.9941	0.9985	0.9969
$NPDV^{Spread}_{db}$	1.0125	0.9737	1.0002	0.9913	1.0015	0.9962

Note: We report ratios of the average RMSE and QLike losses of all RV measures relative to the RV^{5min} measure. A gray-shaded cell indicates that the associated estimate belongs to the 75% model confidence set.

Table 6: Relative forecasting performance for BAC

	1 day ahead		1 week ahead		1 month ahead	
	RMSE	QLIKE	RMSE	QLIKE	RMSE	QLIKE
RV^{5min}	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
RV_{ss}^{5min}	0.9841	1.0014	0.9798	0.9934	1.0025	0.9987
RK_{parzen}^{5sec}	0.9845	0.9998	0.9828	0.9935	1.0050	0.9998
RK_{cubic}^{5sec}	0.9822	1.0054	0.9770	0.9960	1.0054	1.0009
RK_{th2}^{5sec}	0.9844	1.0005	0.9673	0.9941	1.0054	0.9986
$RK_{bartlett}^{5sec}$	0.9850	0.9988	0.9694	0.9938	1.0051	0.9982
$TSRV_{db}^{5sec}$	0.9911	1.0046	0.9599	0.9946	1.0020	0.9885
$MSRV^{5sec}$	0.9878	1.0394	0.9558	0.9885	1.0005	0.9845
RV_{pa}^{5min}	1.0056	1.0499	0.9724	1.0039	1.0032	0.9957
BPV_{db}^{5min}	1.0005	1.0187	1.0220	1.0020	0.9943	1.0042
$BPV_{ss, db}^{5min}$	0.9887	1.0061	0.9753	0.9912	1.0003	0.9963
$PDV_{FHMD, db}^{5min}$	0.9772	0.9969	0.9554	0.9673	1.0081	0.9849
$PDV_{FHMD-Burr,db}^{5min}$	0.9763	0.9955	0.9538	0.9647	1.0088	0.9847
$PDV_{FHMD, db}^{Spread}$	0.9634	0.9693	0.9550	0.9609	1.0200	0.9843
$PDV_{FHMD-Burr,db}^{Spread}$	0.9687	0.9595	0.9545	0.9627	1.0185	0.9841
$PDV_{MSMD, db}^{5min}$	0.9772	0.9957	0.9554	0.9661	1.0080	0.9843
$PDV_{MSMD-Burr,db}^{5min}$	0.9681	0.9922	0.9507	0.9629	1.0085	0.9807
$PDV_{MSMD, db}^{Spread}$	0.9631	0.9739	0.9564	0.9608	1.0208	0.9826
$PDV_{MSMD-Burr,db}^{Spread}$	0.9705	0.9775	0.9524	0.9650	1.0206	0.9840
$PDV_{ACD, db}^{5min}$	0.9828	0.9851	0.9517	0.9707	1.0090	0.9824
$PDV_{ACD-Burr,db}^{5min}$	0.9818	0.9888	0.9526	0.9734	1.0090	0.9831
$PDV_{ACD, db}^{Spread}$	0.9666	0.9207	0.9541	0.9396	1.0198	0.9807
$PDV_{ACD-Burr,db}^{Spread}$	0.9692	0.9334	0.9516	0.9449	1.0196	0.9839
$PDV_{FIACD, db}^{5min}$	1.0563	1.2763	0.9914	1.0039	0.9982	0.9852
$PDV_{FIACD-Burr,db}^{5min}$	1.1287	1.2727	1.0301	1.0010	0.9946	0.9844
$PDV_{FIACD, db}^{Spread}$	0.9803	1.4233	0.9723	1.0931	1.0314	1.0424
$PDV_{FIACD-Burr,db}^{Spread}$	0.9811	1.4018	0.9725	1.0874	1.0308	1.0403
$PDV_{Log-ACD,db}^{5min}$	0.9826	0.9867	0.9526	0.9716	1.0087	0.9828
$PDV_{Log-ACD-Burr,db}^{5min}$	0.9816	0.9910	0.9535	0.9743	1.0087	0.9835
$PDV_{Log-ACD,db}^{Spread}$	0.9654	0.9327	0.9545	0.9437	1.0194	0.9844
$PDV_{Log-ACD-Burr,db}^{Spread}$	0.9698	0.9422	0.9500	0.9449	1.0190	0.9861
$NPDV_{db}^{5min}$	0.9747	0.9949	0.9541	0.9770	1.0087	0.9806
$NPDV_{db}^{Spread}$	0.9574	1.0080	0.9588	0.9743	1.0220	0.9875

Note: Analogous to the notes to Table 5.

Table 7: Relative forecasting performance for DIS

	1 day ahead		1 week ahead		1 month ahead	
	RMSE	QLIKE	RMSE	QLIKE	RMSE	QLIKE
RV^{5min}	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
RV^{5min}_{ss}	1.0110	0.9742	0.9984	0.9870	1.0019	0.9967
RK^{5sec}_{parzen}	1.0122	0.9828	1.0010	0.9920	1.0027	0.9954
RK^{5sec}_{cubic}	1.0051	0.9845	1.0000	0.9939	1.0022	0.9958
RK^{5sec}_{th2}	0.9936	0.9846	0.9978	0.9980	1.0009	0.9951
$RK^{5sec}_{bartlett}$	0.9941	0.9821	0.9978	0.9963	1.0010	0.9956
$TSRV^{5sec}_{db}$	0.9809	1.0019	0.9970	1.0002	1.0002	1.0043
$MSRV^{5sec}$	0.9838	1.0332	0.9978	1.0107	1.0003	1.0064
RV^{5min}_{pa}	1.0098	1.0327	0.9961	0.9774	1.0031	0.9988
BPV^{5min}_{db}	0.9903	0.9664	0.9952	0.9868	1.0023	1.0054
$BPV^{5min}_{ss, db}$	1.0060	0.9559	0.9972	0.9863	1.0030	1.0000
$PDV^{5min}_{FHMD, db}$	0.9735	0.9532	0.9952	0.9889	0.9986	0.9973
$PDV^{5min}_{FHMD-Burr,db}$	0.9731	0.9515	0.9948	0.9885	0.9985	0.9975
$PDV^{Spread}_{FHMD, db}$	0.9508	0.9385	0.9862	0.9893	1.0001	1.0091
$PDV^{Spread}_{FHMD-Burr,db}$	0.9497	0.9335	0.9856	0.9902	0.9998	1.0093
$PDV^{5min}_{MSMD, db}$	0.9734	0.9524	0.9951	0.9892	0.9986	0.9982
$PDV^{5min}_{MSMD-Burr,db}$	0.9773	0.9568	0.9952	0.9879	0.9992	0.9990
$PDV^{Spread}_{MSMD, db}$	0.9522	0.9528	0.9876	0.9960	1.0000	1.0103
$PDV^{Spread}_{MSMD-Burr,db}$	0.9537	0.9469	0.9861	0.9913	1.0003	1.0111
$PDV^{5min}_{ACD, db}$	0.9699	0.9445	0.9953	0.9808	0.9989	0.9977
$PDV^{5min}_{ACD-Burr,db}$	0.9697	0.9437	0.9949	0.9812	0.9988	0.9975
$PDV^{Spread}_{ACD, db}$	0.9409	0.9002	0.9843	0.9769	0.9998	1.0083
$PDV^{Spread}_{ACD-Burr,db}$	0.9429	0.9122	0.9849	0.9795	0.9998	1.0089
$PDV^{5min}_{FIACD, db}$	1.0361	1.2131	0.9994	0.9898	1.0023	1.0289
$PDV^{5min}_{FIACD-Burr,db}$	1.0284	1.1787	0.9982	0.9877	1.0006	1.0270
$PDV^{Spread}_{FIACD, db}$	1.0130	1.0507	1.0030	0.9511	1.0013	0.9913
$PDV^{Spread}_{FIACD-Burr,db}$	1.0103	1.0277	1.0016	0.9540	1.0014	0.9912
$PDV^{5min}_{Log-ACD,db}$	0.9703	0.9460	0.9950	0.9810	0.9989	0.9978
$PDV^{5min}_{Log-ACD-Burr,db}$	0.9702	0.9453	0.9947	0.9813	0.9989	0.9976
$PDV^{Spread}_{Log-ACD,db}$	0.9418	0.9129	0.9839	0.9750	0.9996	1.0091
$PDV^{Spread}_{Log-ACD-Burr,db}$	0.9441	0.9204	0.9840	0.9791	0.9999	1.0104
$NPDV^{5min}_{db}$	0.9741	0.9504	0.9950	0.9832	0.9995	0.9984
$NPDV^{Spread}_{db}$	0.9576	0.9627	0.9901	0.9981	0.9998	1.0067

Note: Analogous to the notes to Table 5.

Table 8: Relative forecasting performance for EVRG

	1 day ahead		1 week ahead		1 month ahead	
	RMSE	QLIKE	RMSE	QLIKE	RMSE	QLIKE
RV^{5min}	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
RV_{ss}^{5min}	0.9960	0.9811	0.9982	0.9925	0.9993	0.9999
RK_{parzen}^{5sec}	0.9930	0.9637	0.9998	0.9863	0.9989	0.9961
RK_{cubic}^{5sec}	0.9960	0.9768	1.0014	0.9979	0.9991	1.0022
RK_{th2}^{5sec}	0.9979	0.9868	1.0009	1.0034	0.9991	1.0040
$RK_{bartlett}^{5sec}$	0.9977	0.9852	1.0008	1.0016	0.9990	1.0035
$TSRV_{db}^{5sec}$	1.0087	1.0173	1.0008	1.0062	0.9999	1.0180
$MSRV^{5sec}$	1.0183	1.0708	1.0059	1.0397	1.0000	1.0249
RV_{pa}^{5min}	0.9942	1.0176	0.9971	0.9987	0.9986	0.9797
BPV_{db}^{5min}	0.9997	0.9570	0.9953	0.9557	0.9974	0.9772
$BPV_{ss, db}^{5min}$	0.9951	0.9405	0.9936	0.9484	0.9976	0.9843
$PDV_{FHMD, db}^{5min}$	1.0022	0.9895	0.9989	0.9929	0.9981	1.0092
$PDV_{FHMD-Burr,db}^{5min}$	1.0031	0.9819	0.9968	0.9865	0.9981	1.0068
$PDV_{FHMD, db}^{Spread}$	1.0015	0.9846	0.9973	0.9673	0.9979	1.0008
$PDV_{FHMD-Burr,db}^{Spread}$	1.0002	0.9826	0.9962	0.9659	0.9977	0.9995
$PDV_{MSMD, db}^{5min}$	1.0029	0.9991	0.9994	0.9994	0.9981	1.0091
$PDV_{MSMD-Burr,db}^{5min}$	1.0025	0.9907	0.9990	0.9946	0.9982	1.0109
$PDV_{MSMD, db}^{Spread}$	1.0007	0.9873	0.9982	0.9740	0.9979	0.9992
$PDV_{MSMD-Burr,db}^{Spread}$	1.0008	0.9826	0.9977	0.9724	0.9980	0.9999
$PDV_{ACD, db}^{5min}$	1.0032	0.9948	0.9985	0.9894	0.9980	1.0084
$PDV_{ACD-Burr,db}^{5min}$	1.0028	0.9921	0.9987	0.9901	0.9980	1.0092
$PDV_{ACD, db}^{Spread}$	0.9977	0.9737	0.9945	0.9441	0.9969	0.9919
$PDV_{ACD-Burr,db}^{Spread}$	0.9974	0.9704	0.9950	0.9502	0.9970	0.9932
$PDV_{FIACD, db}^{5min}$	1.0338	1.1609	1.0101	1.0429	0.9975	1.0110
$PDV_{FIACD-Burr,db}^{5min}$	1.0431	1.2221	1.0157	1.0841	0.9993	1.0205
$PDV_{FIACD, db}^{Spread}$	1.0217	1.1224	1.0050	0.9747	0.9985	1.0228
$PDV_{FIACD-Burr,db}^{Spread}$	1.0215	1.1110	1.0052	0.9825	0.9992	1.0443
$PDV_{Log-ACD,db}^{5min}$	1.0036	0.9947	0.9985	0.9891	0.9980	1.0083
$PDV_{Log-ACD-Burr,db}^{5min}$	1.0031	0.9918	0.9987	0.9899	0.9980	1.0090
$PDV_{Log-ACD,db}^{Spread}$	0.9969	0.9680	0.9945	0.9455	0.9973	0.9959
$PDV_{Log-ACD-Burr,db}^{Spread}$	0.9967	0.9666	0.9952	0.9516	0.9972	0.9947
$NPDV_{db}^{5min}$	1.0039	0.9976	0.9983	0.9951	0.9982	1.0090
$NPDV_{db}^{Spread}$	1.0006	0.9893	0.9986	0.9821	0.9980	0.9985

Note: Analogous to the notes to Table 5.

Table 9: Relative forecasting performance for FB

	1 day ahead		1 week ahead		1 month ahead	
	RMSE	QLIKE	RMSE	QLIKE	RMSE	QLIKE
RV^{5min}	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
RV_{ss}^{5min}	0.9954	1.0033	0.9996	1.0031	1.0006	1.0127
RK_{parzen}^{5sec}	0.9933	0.9944	0.9989	0.9980	1.0002	1.0075
RK_{cubic}^{5sec}	0.9929	0.9909	0.9989	0.9967	1.0003	1.0062
RK_{th2}^{5sec}	0.9927	0.9856	0.9987	0.9947	1.0000	1.0042
$RK_{bartlett}^{5sec}$	0.9929	0.9873	0.9988	0.9960	1.0001	1.0057
$TSRV_{db}^{5sec}$	1.0037	1.0186	1.0006	1.0031	1.0014	1.0107
$MSRV^{5sec}$	1.0037	1.0189	1.0007	1.0034	1.0015	1.0109
RV_{pa}^{5min}	0.9975	1.0635	0.9976	1.0174	1.0017	1.0307
BPV_{db}^{5min}	0.9991	1.0215	1.0001	1.0161	1.0004	1.0108
$BPV_{ss, db}^{5min}$	0.9958	1.0103	0.9994	1.0061	1.0004	1.0150
$PDV_{FHMD, db}^{5min}$	0.9978	1.0000	1.0003	0.9992	1.0016	1.0118
$PDV_{FHMD-Burr,db}^{5min}$	0.9907	0.9979	1.0000	1.0040	1.0006	1.0126
$PDV_{FHMD, db}^{Spread}$	0.9895	1.0041	0.9979	0.9917	0.9991	1.0060
$PDV_{FHMD-Burr,db}^{Spread}$	0.9870	0.9954	0.9975	0.9907	0.9992	1.0075
$PDV_{MSMD, db}^{5min}$	0.9915	1.0017	1.0001	1.0040	1.0006	1.0139
$PDV_{MSMD-Burr,db}^{5min}$	0.9906	0.9987	0.9998	1.0055	1.0004	1.0115
$PDV_{MSMD, db}^{Spread}$	0.9907	1.0178	0.9990	1.0016	0.9997	1.0114
$PDV_{MSMD-Burr,db}^{Spread}$	0.9886	1.0128	0.9981	0.9976	0.9991	1.0088
$PDV_{ACD, db}^{5min}$	0.9886	1.0017	0.9994	1.0008	0.9989	1.0045
$PDV_{ACD-Burr,db}^{5min}$	0.9886	1.0012	0.9996	1.0013	0.9990	1.0050
$PDV_{ACD, db}^{Spread}$	0.9859	0.9877	0.9972	0.9889	0.9991	1.0062
$PDV_{ACD-Burr,db}^{Spread}$	0.9867	0.9903	0.9974	0.9905	0.9992	1.0072
$PDV_{FIACD, db}^{5min}$	1.0087	1.2309	1.0061	1.0366	1.0009	1.0172
$PDV_{FIACD-Burr,db}^{5min}$	1.0098	1.1750	1.0022	1.0122	0.9975	0.9937
$PDV_{FIACD, db}^{Spread}$	0.9869	0.9974	0.9961	0.9682	0.9985	1.0077
$PDV_{FIACD-Burr,db}^{Spread}$	0.9862	0.9926	0.9958	0.9670	0.9983	1.0052
$PDV_{Log-ACD,db}^{5min}$	0.9887	1.0019	0.9995	1.0018	0.9990	1.0053
$PDV_{Log-ACD-Burr,db}^{5min}$	0.9887	1.0015	0.9997	1.0021	0.9990	1.0056
$PDV_{Log-ACD,db}^{Spread}$	0.9867	0.9883	0.9973	0.9894	0.9991	1.0061
$PDV_{Log-ACD-Burr,db}^{Spread}$	0.9880	0.9966	0.9972	0.9886	0.9986	1.0032
$NPDV_{db}^{5min}$	0.9888	1.0006	1.0001	1.0056	0.9994	1.0088
$NPDV_{db}^{Spread}$	0.9897	1.0186	0.9994	1.0044	0.9996	1.0114

Note: Analogous to the notes to Table 5.

Table 10: Relative forecasting performance for GE

	1 day ahead		1 week ahead		1 month ahead	
	RMSE	QLIKE	RMSE	QLIKE	RMSE	QLIKE
RV^{5min}	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
RV_{ss}^{5min}	1.0045	0.9983	0.9972	1.0137	1.0029	1.0155
RK_{parzen}^{5sec}	1.0027	1.0164	0.9982	1.0178	1.0031	1.0179
RK_{cubic}^{5sec}	1.0002	1.0302	0.9950	1.0269	1.0038	1.0255
RK_{th2}^{5sec}	0.9874	1.0172	0.9908	1.0349	1.0053	1.0329
$RK_{bartlett}^{5sec}$	0.9884	1.0140	0.9909	1.0321	1.0050	1.0320
$TSRV_{db}^{5sec}$	0.9706	0.9914	0.9856	1.0397	1.0081	1.0346
$MSRV^{5sec}$	0.9690	1.0029	0.9821	1.0274	1.0046	0.9866
RV_{pa}^{5min}	1.0041	1.1038	1.0101	1.0873	1.0136	1.1299
BPV_{db}^{5min}	0.9978	1.0037	1.0053	1.0064	1.0038	1.0172
$BPV_{ss, db}^{5min}$	0.9989	0.9921	0.9977	1.0154	1.0046	1.0273
$PDV_{FHMD, db}^{5min}$	0.9687	0.9658	0.9860	1.0152	1.0044	1.0122
$PDV_{FHMD-Burr,db}^{5min}$	0.9701	0.9641	0.9866	1.0155	1.0046	1.0118
$PDV_{FHMD, db}^{Spread}$	0.9266	0.9471	0.9633	0.9631	0.9999	0.9980
$PDV_{FHMD-Burr,db}^{Spread}$	0.9261	0.9381	0.9645	0.9555	0.9997	0.9966
$PDV_{MSMD, db}^{5min}$	0.9692	0.9664	0.9860	1.0160	1.0042	1.0115
$PDV_{MSMD-Burr,db}^{5min}$	0.9719	0.9774	0.9881	1.0263	1.0048	1.0181
$PDV_{MSMD, db}^{Spread}$	0.9238	0.9584	0.9677	0.9767	1.0012	1.0062
$PDV_{MSMD-Burr,db}^{Spread}$	0.9234	0.9500	0.9649	0.9716	0.9998	1.0015
$PDV_{ACD, db}^{5min}$	0.9751	0.9533	0.9875	1.0107	1.0053	1.0119
$PDV_{ACD-Burr,db}^{5min}$	0.9753	0.9574	0.9873	1.0110	1.0053	1.0132
$PDV_{ACD, db}^{Spread}$	0.9107	0.9200	0.9576	0.9451	0.9966	0.9923
$PDV_{ACD-Burr,db}^{Spread}$	0.9101	0.9313	0.9573	0.9492	0.9975	0.9965
$PDV_{FIACD, db}^{5min}$	1.0918	1.1505	1.0271	1.0865	1.0008	1.0643
$PDV_{FIACD-Burr,db}^{5min}$	1.0700	1.1909	1.0166	1.1128	1.0042	1.0650
$PDV_{FIACD, db}^{Spread}$	0.9960	1.3640	0.9765	1.2513	1.0141	1.3323
$PDV_{FIACD-Burr,db}^{Spread}$	0.9947	1.3941	0.9785	1.2864	1.0147	1.3500
$PDV_{Log-ACD,db}^{5min}$	0.9754	0.9557	0.9876	1.0116	1.0054	1.0125
$PDV_{Log-ACD-Burr,db}^{5min}$	0.9754	0.9596	0.9873	1.0116	1.0053	1.0138
$PDV_{Log-ACD,db}^{Spread}$	0.9162	0.9293	0.9581	0.9518	0.9970	0.9960
$PDV_{Log-ACD-Burr,db}^{Spread}$	0.9163	0.9406	0.9564	0.9507	0.9963	0.9931
$NPDV_{db}^{5min}$	0.9754	0.9581	0.9882	1.0163	1.0058	1.0156
$NPDV_{db}^{Spread}$	0.9336	0.9922	0.9661	0.9966	1.0002	1.0110

Note: Analogous to the notes to Table 5.

Table 11: Relative forecasting performance for IBM

	1 day ahead		1 week ahead		1 month ahead	
	RMSE	QLIKE	RMSE	QLIKE	RMSE	QLIKE
RV^{5min}	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$\text{RV}^{\text{5min}}_{\text{ss}}$	1.0194	0.9603	1.0013	0.9969	1.0006	0.9992
$\text{RK}^{\text{5sec}}_{\text{parzen}}$	1.0242	0.9647	1.0032	1.0020	1.0009	0.9993
$\text{RK}^{\text{5sec}}_{\text{cubic}}$	1.0182	0.9593	1.0022	1.0055	1.0008	0.9964
$\text{RK}^{\text{5sec}}_{\text{th2}}$	1.0021	0.9611	0.9999	1.0039	1.0007	0.9960
$\text{RK}^{\text{5sec}}_{\text{bartlett}}$	1.0019	0.9579	1.0000	1.0039	1.0006	0.9963
$\text{TSRV}^{\text{5sec}}_{\text{db}}$	1.0017	0.9703	1.0054	1.0104	1.0011	1.0035
$\text{MSRV}^{\text{5sec}}$	1.0034	0.9911	1.0067	1.0218	1.0016	1.0100
$\text{RV}^{\text{5min}}_{\text{pa}}$	1.0084	1.0119	1.0015	1.0084	0.9995	0.9927
$\text{BPV}^{\text{5min}}_{\text{db}}$	0.9972	0.9872	0.9994	0.9913	0.9993	0.9938
$\text{BPV}^{\text{5min}}_{\text{ss, db}}$	1.0203	0.9539	1.0023	0.9954	1.0005	0.9953
$\text{PDV}^{\text{5min}}_{\text{FHMD, db}}$	1.0041	0.9573	1.0025	1.0092	1.0016	1.0030
$\text{PDV}^{\text{5min}}_{\text{FHMD-Burr,db}}$	0.9999	0.9553	1.0017	1.0080	1.0014	1.0030
$\text{PDV}^{\text{Spread}}_{\text{FHMD, db}}$	0.9940	0.9382	1.0022	1.0035	1.0019	1.0101
$\text{PDV}^{\text{Spread}}_{\text{FHMD-Burr,db}}$	0.9934	0.9367	1.0018	0.9995	1.0019	1.0094
$\text{PDV}^{\text{5min}}_{\text{MSMD, db}}$	0.9937	0.9554	1.0017	1.0093	1.0013	1.0032
$\text{PDV}^{\text{5min}}_{\text{MSMD-Burr,db}}$	0.9997	0.9616	1.0016	1.0124	1.0017	1.0045
$\text{PDV}^{\text{Spread}}_{\text{MSMD, db}}$	0.9978	0.9556	1.0041	1.0085	1.0022	1.0120
$\text{PDV}^{\text{Spread}}_{\text{MSMD-Burr,db}}$	0.9973	0.9456	1.0028	1.0006	1.0014	1.0063
$\text{PDV}^{\text{5min}}_{\text{ACD, db}}$	0.9908	0.9571	1.0021	1.0066	1.0016	1.0036
$\text{PDV}^{\text{5min}}_{\text{ACD-Burr,db}}$	0.9915	0.9576	1.0022	1.0065	1.0016	1.0039
$\text{PDV}^{\text{Spread}}_{\text{ACD, db}}$	0.9856	0.9121	0.9992	0.9853	1.0008	1.0046
$\text{PDV}^{\text{Spread}}_{\text{ACD-Burr,db}}$	0.9868	0.9207	1.0003	0.9896	1.0009	1.0054
$\text{PDV}^{\text{5min}}_{\text{FIACD, db}}$	1.0329	1.1749	1.0210	1.0806	1.0040	1.0272
$\text{PDV}^{\text{5min}}_{\text{FIACD-Burr,db}}$	1.0270	1.1255	1.0170	1.0574	1.0031	1.0140
$\text{PDV}^{\text{Spread}}_{\text{FIACD, db}}$	1.0281	1.1503	1.0111	1.0439	1.0025	1.0235
$\text{PDV}^{\text{Spread}}_{\text{FIACD-Burr,db}}$	1.0220	1.1164	1.0104	1.0371	1.0025	1.0266
$\text{PDV}^{\text{5min}}_{\text{Log-ACD,db}}$	0.9916	0.9577	1.0024	1.0072	1.0016	1.0036
$\text{PDV}^{\text{5min}}_{\text{Log-ACD-Burr,db}}$	0.9924	0.9586	1.0025	1.0070	1.0015	1.0040
$\text{PDV}^{\text{Spread}}_{\text{Log-ACD,db}}$	0.9866	0.9146	0.9997	0.9879	1.0009	1.0052
$\text{PDV}^{\text{Spread}}_{\text{Log-ACD-Burr,db}}$	0.9869	0.9209	1.0000	0.9891	1.0008	1.0058
$\text{NPDV}^{\text{5min}}_{\text{db}}$	0.9929	0.9537	1.0028	1.0109	1.0018	1.0037
$\text{NPDV}^{\text{Spread}}_{\text{db}}$	0.9936	0.9458	1.0035	1.0095	1.0013	1.0052

Note: Analogous to the notes to Table 5.

Table 12: Relative forecasting performance for PFE

	1 day ahead		1 week ahead		1 month ahead	
	RMSE	QLIKE	RMSE	QLIKE	RMSE	QLIKE
RV^{5min}	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
RV^{5min}_{ss}	0.9992	0.9808	0.9997	1.0051	0.9988	0.9995
RK^{5sec}_{parzen}	0.9982	0.9718	0.9993	1.0014	0.9981	0.9975
RK^{5sec}_{cubic}	1.0038	0.9710	0.9998	1.0079	0.9973	0.9976
RK^{5sec}_{th2}	1.0061	0.9759	0.9996	1.0090	0.9967	0.9962
$RK^{5sec}_{bartlett}$	1.0055	0.9756	0.9999	1.0096	0.9968	0.9972
$TSRV^{5sec}_{db}$	1.0082	1.0039	1.0041	1.0198	0.9981	1.0015
$MSRV^{5sec}$	1.0151	1.0826	1.0060	1.0615	0.9990	1.0213
RV^{5min}_{pa}	0.9977	0.9975	1.0037	1.0029	0.9991	0.9888
BPV^{5min}_{db}	1.0022	1.0019	1.0018	0.9991	0.9999	1.0013
$BPV^{5min}_{ss, db}$	1.0006	0.9811	0.9994	1.0011	0.9987	1.0016
$PDV^{5min}_{FHMD, db}$	0.9852	0.9845	0.9975	1.0167	0.9979	1.0075
$PDV^{5min}_{FHMD-Burr,db}$	0.9860	0.9822	0.9972	1.0136	0.9976	1.0065
$PDV^{Spread}_{FHMD, db}$	0.9708	0.9611	0.9937	1.0163	0.9983	1.0087
$PDV^{Spread}_{FHMD-Burr,db}$	0.9678	0.9486	0.9916	1.0057	0.9981	1.0062
$PDV^{5min}_{MSMD, db}$	0.9848	0.9815	0.9972	1.0145	0.9978	1.0057
$PDV^{5min}_{MSMD-Burr,db}$	0.9883	0.9870	0.9975	1.0148	0.9984	1.0086
$PDV^{Spread}_{MSMD, db}$	0.9687	0.9566	0.9931	1.0157	0.9990	1.0097
$PDV^{Spread}_{MSMD-Burr,db}$	0.9673	0.9532	0.9932	1.0116	0.9988	1.0053
$PDV^{5min}_{ACD, db}$	0.9832	0.9900	0.9991	1.0216	0.9986	1.0116
$PDV^{5min}_{ACD-Burr,db}$	0.9825	0.9899	0.9989	1.0212	0.9988	1.0131
$PDV^{Spread}_{ACD, db}$	0.9491	0.8851	0.9891	0.9835	0.9978	1.0068
$PDV^{Spread}_{ACD-Burr,db}$	0.9544	0.9098	0.9892	0.9930	0.9981	1.0105
$PDV^{5min}_{FIACD, db}$	1.0271	1.1256	1.0123	1.0782	1.0032	1.0414
$PDV^{5min}_{FIACD-Burr,db}$	1.0267	1.1051	1.0103	1.0762	1.0014	1.0405
$PDV^{Spread}_{FIACD, db}$	1.0963	1.3478	1.0305	1.0518	1.0041	0.9704
$PDV^{Spread}_{FIACD-Burr,db}$	1.1082	1.3484	1.0419	1.0540	1.0053	0.9663
$PDV^{5min}_{Log-ACD,db}$	0.9831	0.9892	0.9993	1.0220	0.9986	1.0119
$PDV^{5min}_{Log-ACD-Burr,db}$	0.9824	0.9891	0.9990	1.0216	0.9987	1.0133
$PDV^{Spread}_{Log-ACD,db}$	0.9543	0.9004	0.9915	0.9948	0.9980	1.0077
$PDV^{Spread}_{Log-ACD-Burr,db}$	0.9587	0.9284	0.9901	1.0048	0.9987	1.0102
$NPDV^{5min}_{db}$	0.9859	0.9909	0.9984	1.0201	0.9989	1.0122
$NPDV^{Spread}_{db}$	0.9737	0.9918	0.9969	1.0296	0.9992	1.0115

Note: Analogous to the notes to Table 5.

Table 13: Relative forecasting performance for TSLA

	1 day ahead		1 week ahead		1 month ahead	
	RMSE	QLIKE	RMSE	QLIKE	RMSE	QLIKE
RV^{5min}	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
RV_{ss}^{5min}	1.0043	0.9855	1.0011	0.9950	0.9995	0.9973
RK_{parzen}^{5sec}	1.0069	0.9933	1.0013	0.9939	0.9995	0.9969
RK_{cubic}^{5sec}	1.0127	1.0055	1.0013	0.9971	0.9999	1.0012
RK_{th2}^{5sec}	1.0137	1.0131	1.0034	1.0064	1.0005	1.0026
$RK_{bartlett}^{5sec}$	1.0132	1.0130	1.0031	1.0056	1.0005	1.0035
$TSRV_{db}^{5sec}$	1.0178	1.0469	1.0049	1.0223	1.0018	1.0142
$MSRV^{5sec}$	1.0488	1.1912	1.0188	1.0875	1.0046	1.0319
RV_{pa}^{5min}	1.0252	1.0247	1.0105	1.0280	0.9991	0.9969
BPV_{db}^{5min}	1.0007	0.9935	0.9966	0.9889	1.0003	1.0042
$BPV_{ss, db}^{5min}$	0.9977	0.9781	0.9987	0.9921	1.0007	1.0056
$PDV_{FHMD, db}^{5min}$	0.9955	0.9739	0.9969	0.9889	1.0005	1.0073
$PDV_{FHMD-Burr,db}^{5min}$	0.9952	0.9729	0.9971	0.9895	1.0003	1.0060
$PDV_{FHMD, db}^{Spread}$	0.9965	0.9844	0.9953	0.9852	1.0007	1.0049
$PDV_{FHMD-Burr,db}^{Spread}$	0.9972	0.9869	0.9952	0.9845	1.0005	1.0030
$PDV_{MSMD, db}^{5min}$	0.9954	0.9741	0.9968	0.9889	1.0005	1.0074
$PDV_{MSMD-Burr,db}^{5min}$	0.9976	0.9722	0.9961	0.9810	0.9996	0.9989
$PDV_{MSMD, db}^{Spread}$	1.0009	0.9928	0.9968	0.9899	1.0007	1.0043
$PDV_{MSMD-Burr,db}^{Spread}$	0.9999	0.9886	0.9963	0.9891	1.0005	1.0029
$PDV_{ACD, db}^{5min}$	0.9898	0.9713	0.9949	0.9823	1.0005	1.0087
$PDV_{ACD-Burr,db}^{5min}$	0.9882	0.9715	0.9954	0.9844	1.0007	1.0109
$PDV_{ACD, db}^{Spread}$	0.9877	0.9662	0.9914	0.9755	1.0010	1.0052
$PDV_{ACD-Burr,db}^{Spread}$	0.9877	0.9649	0.9936	0.9793	1.0010	1.0052
$PDV_{FIACD, db}^{5min}$	1.0204	1.1333	1.0047	1.0290	1.0034	1.0458
$PDV_{FIACD-Burr,db}^{5min}$	1.0202	1.1164	1.0084	1.0293	1.0023	1.0237
$PDV_{FIACD, db}^{Spread}$	0.9970	1.0135	0.9857	0.9328	0.9951	0.9793
$PDV_{FIACD-Burr,db}^{Spread}$	1.0016	1.0164	0.9873	0.9337	0.9922	0.9668
$PDV_{Log-ACD,db}^{5min}$	0.9897	0.9715	0.9950	0.9827	1.0005	1.0086
$PDV_{Log-ACD-Burr,db}^{5min}$	0.9882	0.9717	0.9955	0.9847	1.0007	1.0107
$PDV_{Log-ACD,db}^{Spread}$	0.9876	0.9665	0.9913	0.9751	1.0011	1.0055
$PDV_{Log-ACD-Burr,db}^{Spread}$	0.9876	0.9647	0.9930	0.9768	1.0008	1.0028
$NPDV_{db}^{5min}$	0.9931	0.9723	0.9959	0.9853	1.0003	1.0084
$NPDV_{db}^{Spread}$	0.9997	0.9924	0.9976	0.9950	1.0014	1.0112

Note: Analogous to the notes to Table 5.

Table 14: Relative forecasting performance for WMT

	1 day ahead		1 week ahead		1 month ahead	
	RMSE	QLIKE	RMSE	QLIKE	RMSE	QLIKE
RV^{5min}	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
RV_{ss}^{5min}	0.9982	0.9786	0.9991	0.9912	0.9998	0.9967
RK_{parzen}^{5sec}	0.9969	0.9748	0.9982	0.9858	0.9995	0.9938
RK_{cubic}^{5sec}	0.9980	0.9767	0.9981	0.9854	0.9991	0.9903
RK_{th2}^{5sec}	0.9971	0.9780	0.9981	0.9865	0.9987	0.9874
$RK_{bartlett}^{5sec}$	0.9968	0.9745	0.9980	0.9849	0.9987	0.9871
$TSRV_{db}^{5sec}$	1.0018	1.0088	1.0000	0.9899	0.9993	0.9856
$MSRV^{5sec}$	1.0024	1.0105	1.0003	0.9924	0.9994	0.9854
RV_{pa}^{5min}	0.9998	1.0056	1.0013	0.9950	1.0007	1.0001
BPV_{db}^{5min}	0.9974	0.9834	0.9997	0.9979	0.9999	0.9962
$BPV_{ss, db}^{5min}$	0.9969	0.9634	0.9988	0.9895	0.9998	0.9963
$PDV_{FHMD, db}^{5min}$	0.9966	0.9707	0.9980	0.9809	0.9988	0.9826
$PDV_{FHMD-Burr,db}^{5min}$	0.9964	0.9703	0.9979	0.9801	0.9988	0.9830
$PDV_{FHMD, db}^{Spread}$	0.9901	0.9431	0.9939	0.9502	0.9987	0.9837
$PDV_{FHMD-Burr,db}^{Spread}$	0.9888	0.9185	0.9931	0.9450	0.9986	0.9830
$PDV_{MSMD, db}^{5min}$	0.9968	0.9708	0.9981	0.9808	0.9987	0.9825
$PDV_{MSMD-Burr,db}^{5min}$	0.9990	0.9790	0.9987	0.9815	0.9990	0.9834
$PDV_{MSMD, db}^{Spread}$	0.9882	0.9233	0.9940	0.9540	0.9991	0.9862
$PDV_{MSMD-Burr,db}^{Spread}$	0.9885	0.9267	0.9938	0.9505	0.9990	0.9841
$PDV_{ACD, db}^{5min}$	0.9958	0.9698	0.9975	0.9781	0.9985	0.9842
$PDV_{ACD-Burr,db}^{5min}$	0.9961	0.9679	0.9976	0.9796	0.9986	0.9844
$PDV_{ACD, db}^{Spread}$	0.9791	0.8760	0.9896	0.9185	0.9977	0.9739
$PDV_{ACD-Burr,db}^{Spread}$	0.9824	0.8846	0.9913	0.9295	0.9981	0.9773
$PDV_{FIACD, db}^{5min}$	1.0114	1.0877	1.0020	1.0075	0.9983	0.9767
$PDV_{FIACD-Burr,db}^{5min}$	1.0156	1.0920	1.0056	1.0147	0.9994	0.9860
$PDV_{FIACD, db}^{Spread}$	1.0197	1.1537	0.9988	0.9737	1.0004	1.0037
$PDV_{FIACD-Burr,db}^{Spread}$	1.0136	1.1121	0.9963	0.9524	0.9998	0.9947
$PDV_{Log-ACD,db}^{5min}$	0.9959	0.9694	0.9975	0.9785	0.9986	0.9845
$PDV_{Log-ACD-Burr,db}^{5min}$	0.9962	0.9673	0.9977	0.9800	0.9987	0.9846
$PDV_{Log-ACD,db}^{Spread}$	0.9807	0.8962	0.9896	0.9166	0.9980	0.9770
$PDV_{Log-ACD-Burr,db}^{Spread}$	0.9828	0.8879	0.9911	0.9294	0.9982	0.9787
$NPDV_{db}^{5min}$	0.9970	0.9729	0.9983	0.9837	0.9988	0.9859
$NPDV_{db}^{Spread}$	0.9917	0.9580	0.9964	0.9784	0.9989	0.9863

Note: Analogous to the notes to Table 5.

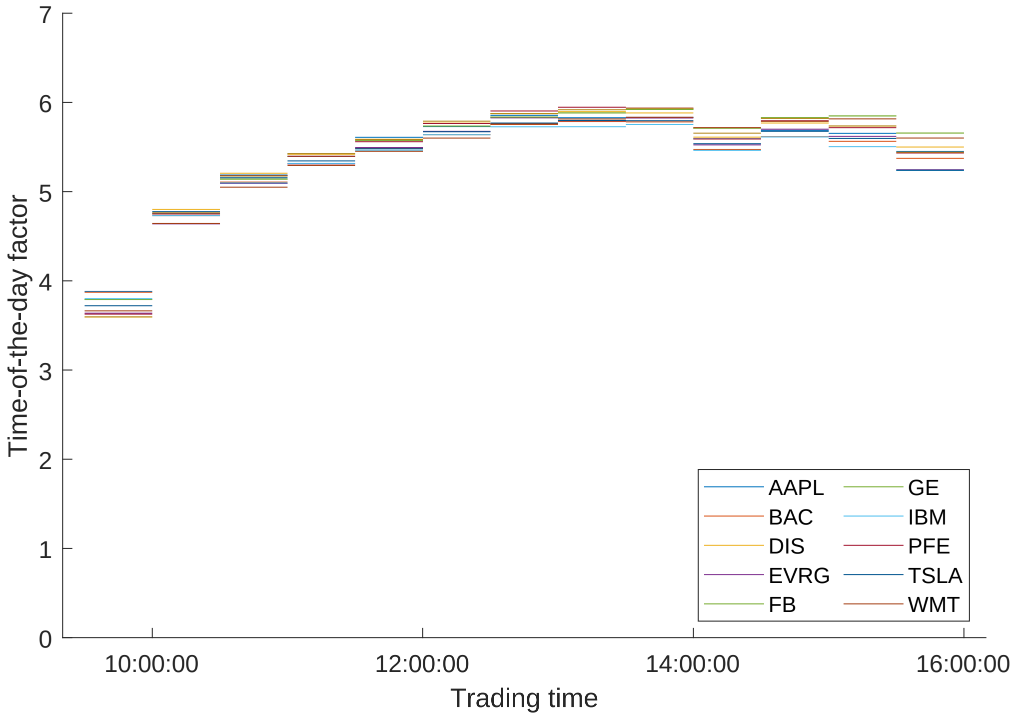

Figure 1: Plots of the mean diurnal seasonality factor for all ten stocks using 5 min. data.

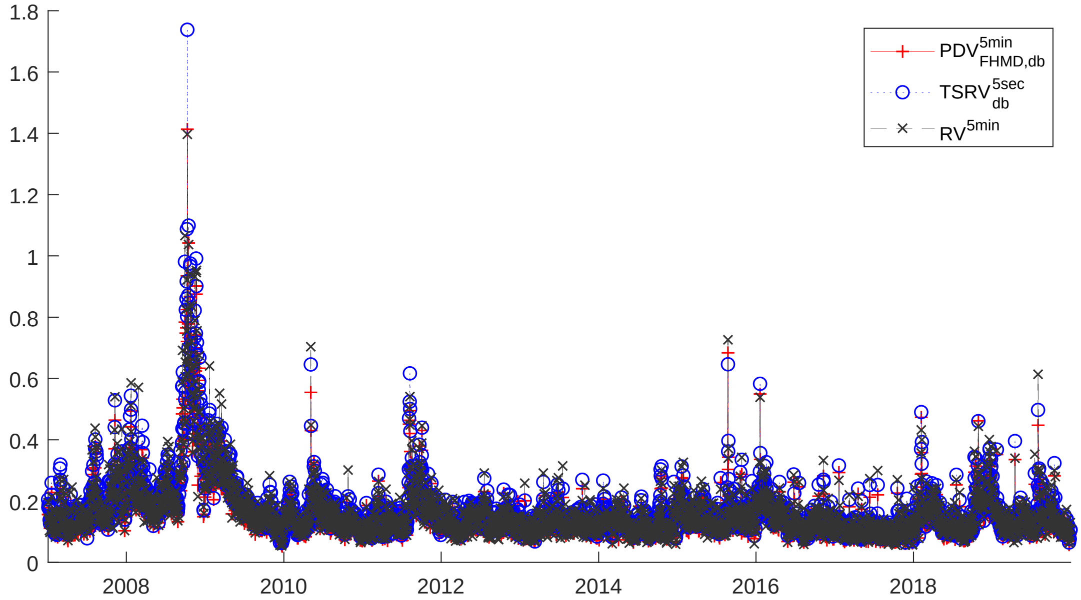

Figure 2: Annualized volatility of IBM

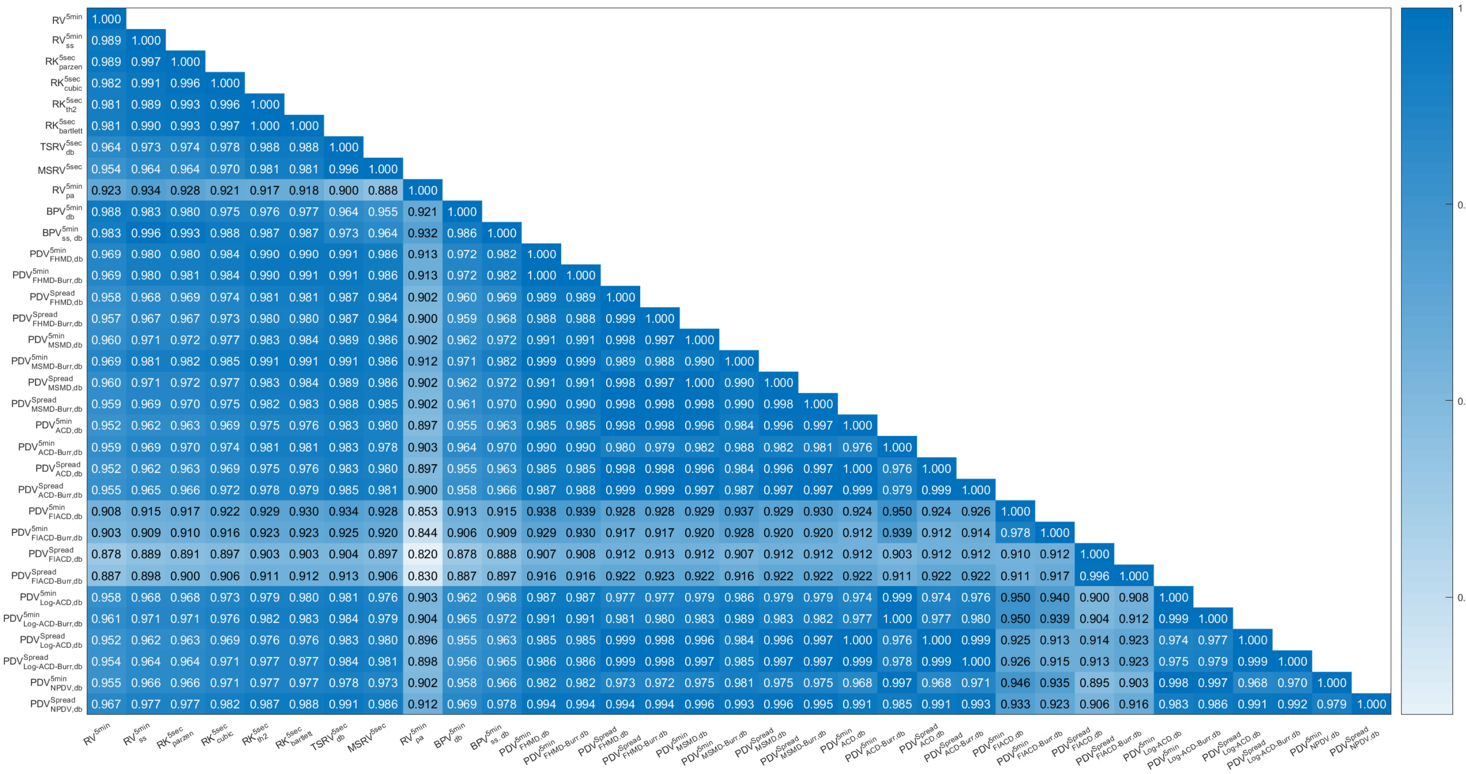

Figure 3: Correlation heatmap for realized variance estimates (IBM)

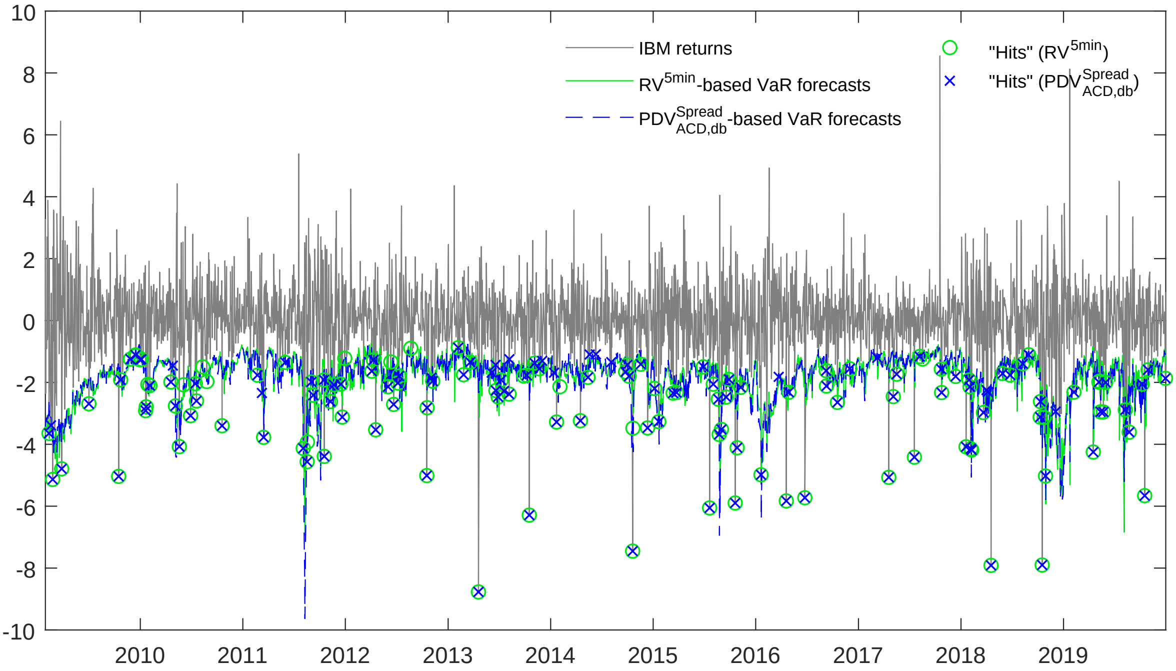

Figure 4: 5%-VaR forecasts for IBM returns using RV^{5min} and $\text{PDV}^{\text{Spread}}_{\text{ACD,db}}$.