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Price Discovery in Thinly Traded Futures Markets: How Thin is Too Thin?

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Abstract

It is still an unanswered question how much trading activity is needed for efficient price discovery in commodity futures markets. For this purpose, we investigate the price discovery process of two thinly traded agricultural futures contracts traded at the European Exchange in Frankfurt. Our empirical results show that the trading volume threshold which is necessary to facilitate efficient price discovery is very low. As our findings are based on constant and time-varying vector error correction models, we also show that neglecting time-variation in the parameters can lead to misleading results, especially for thinly traded markets.

JEL Classification: G12, G13, Q14

Keywords: Thinly Traded Markets, Price Discovery, Trading Volume Threshold, Information Shares, Kalman Filter

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1 Introduction

One major benefit attributed to futures trading is the facilitation of price discovery. Theoretical arguments suggest that futures markets react quicker to new information than their underlying spot markets due to advantages such as higher liquidity, more transparency, and lower transaction costs (Working 1962; Black 1976).

The assumption of liquid futures markets, however, is not always met in practice. For example, Silber (1981) documents that up to 75 percent of new futures contracts fail to attract a profitable level of trading volume. Data from September 2014 support these findings: trading statistics for the three largest futures exchanges in North America and Europe (CME Group, Eurex, ICE) reveal that between 58 and 68 percent of active contracts had a monthly trading volume of less than 10,000 transactions, which Holder et al. (1999) regard as the key criterion to define a futures contract as successful.

Given that low trading volume also implies less available information on the market, the absence of sufficient liquidity may have detrimental effects on the price discovery process (Tomek 1980). In this context, the question whether thinly traded futures markets efficiently fulfill their price discovery function is relevant. Although Garbade and Silber (1983) are among the first to hypothesize that limited trading activity can lead to poor price discovery in futures markets, a systematic empirical approach to this question is still missing in the literature.

This paper aims to fill this gap by investigating two agricultural futures contracts, hog and piglet, which are traded at the Eurex in Frankfurt. We choose these contracts due to their exceptionally low trading volume since their introduction in July 2009. The average number of monthly transactions amounted to 147 for the hog contract and 28 for the piglet contract between July 2009 and April 2014. For comparison, the lean hog futures contract traded at the CME reached a monthly trading volume of about one million since mid-2009. Our sample thus consists of one extreme case of a thinly traded market (piglet) and a more moderate one (hog). By comparing the empirical results of these two markets, we intend to obtain insight into the link between trading activity and price discovery.

Our empirical approaches are based on cointegration techniques and on vector error correction models (VECMs). To test for cointegration we use the autoregressive distributed lag (ARDL) bounds test proposed by Pesaran and Shin (1999) and Pesaran et al. (2001) due to the low power of unit-root tests in shorter samples. This is given in our paper as the data consists of weekly prices for a rather short time period. To examine the relative contribution of each market to the price discovery process, we calculate three price discovery measures that are frequently applied in the literature. Since structural changes and temporal shocks are often not revealed in these econometric models, we also account for time-variation in the parameters by applying the Kalman filter. This econometric technique additionally distinguishes our paper from previous studies, which predominantly use time-invariant approaches and hence only provide indications of average price discovery.

The common price discovery measures suggest that both futures markets are dominant in the price discovery process. By estimating time-varying parameters, however, we reveal that price discovery in the less liquid piglet market is characterized by strong fluctuations. Comparing the results of both markets leads us to conclude that the trading volume threshold which is necessary to facilitate efficient price discovery is very low. In addition, the findings emphasize the importance of using empirical models that account for time-varying parameters, particularly when analyzing thinly traded futures markets.

Our findings are relevant for producers and traders of physical commodities who may question the quality of price signals from futures markets with little trading activity. Our empirical results indicate that even some dozen transactions per week may suffice to generate reliable price information for production and storage decisions. Our study also serves to attenuate concerns among regulators who challenge the efficiency gains associated with the introduction of new commodity futures contracts.

The remainder of the paper is structured as follows. In Section 2 we provide an overview of the related literature. We then outline the empirical approach in Section 3. Section 4 presents the data and Section 5 discusses the empirical results. Section 6 summarizes our findings and concludes.

2 Literature Review

According to financial theory, the possibility of arbitrage should prevent spot and futures prices of the same asset from drifting apart over time.[1] The empirical implication of this, namely that spot and futures prices are cointegrated, has been tested for various asset classes. In general, the majority of this empirical literature supports the hypothesis of an equilibrium relationship between the two price series. For example, cointegration has been found between spot and futures of storable (e.g., crude oil, corn) and non-storable (e.g., live cattle, hogs) commodities (Yang et al. 2001; Maslyuk and Smyth 2009), foreign exchange rates (Kroner and Sultan 1993) and stock indices (Brooks et al. 2001). A few empirical studies, however, fail to identify a cointegrating relationship between spot and futures prices, wherein the lack of sufficient trading activity is often put forward as a possible explanation for this result (e.g., Fortenberry and Zapata 1997; Mattos and Garcia 2004).

The empirical evidence regarding the lead-lag relationships, i.e., the price discovery process, also supports the hypothesis that price discovery is dominated by futures trading, e.g., in equity markets (Hasbrouck 2003; Zhong et al. 2004; Chou and Chung 2006), markets for carbon emissions (Rittler 2012; Mizrach and Otsubo 2014), and commodity markets (Kuiper et al. 2002; Peri et al. 2013; Shrestha 2014). For the latter, empirical research documents no systematic difference in the price discovery performance between storable and non-storable commodities (Yang et al. 2001; Garcia and Leuthold 2004).

The findings of the empirical literature, however, are mixed with regards to thinly traded

[1] More specifically, the cost-of-carry model relates the existence of an equilibrium relationship between spot and futures prices to the cost and benefits of holding the physical asset relative to the futures contract. In this framework, costs of holding an asset accrue from forgone interest yields and from storing the physical good. These expenses can be saved by holding a futures contract instead. The benefits of holding the physical good arise in the form of a convenience yield resulting from the opportunity to adjust flexibly to unexpected fluctuations in supply and demand conditions (see Brenner and Kroner (1995) for more details on the cost-of-carry concept).

futures markets. Garbade and Silber (1983) are among the first authors to present evidence for a negative influence of low trading volume on the price discovery function of futures markets. Their analysis of four U.S. agricultural commodities identifies a dominance of the futures market relative to the spot market for the three most liquid futures markets but fails to find such a dominance with regards to the less liquid market. Other empirical studies that document a weaker price discovery function for less liquid futures markets include Brockman and Tse (1995) for a set of Canadian agricultural commodities, Mattos and Garcia (2004) for spot and futures prices in Brazilian agricultural markets, and Figuerola-Ferretti and Gonzalo (2010) for a group of metals traded at the London Metal Exchange. Although these results indicate a likely adverse effect of low trading volume on price discovery, none of these studies systematically targets thinly traded futures markets to examine the role of trading activity for price transmissions.

Another aspect which is largely neglected in the literature is the possibility of time variation in the lead-lag relationship between spot and futures prices. Although Silvério and Szklo (2012) and Caporale et al. (2014) study the dynamic evolution of price discovery, they focus on liquid futures markets for crude oil.[2] Yet in thinly traded markets participants cannot buy or sell futures contracts frequently. This may lead to price movements which are unrelated to economic fundamentals (Black 1976), and hence influences the price discovery process. Consequently, examining time dynamics may be important when the focus shifts to thinly traded futures markets. Our study contributes to the literature by investigating time-variation of price discovery in two commodity futures markets characterized by little trading activity.

3 Econometric Methodology

We conduct the empirical analysis in several steps. First, we test for cointegration between spot and futures prices. Thereafter, we estimate a constant-parameter VECM to determine

[2] In addition, Yang et al. (2012) account for time-variation in their analysis of actively traded stock index futures in China.

which market leads *on average* in processing new information. To quantify the relative contribution of each market to the pricing process, we compute three price discovery measures that are commonly applied in the literature. The last step consists of reformulating the VECM in state-space form and applying the Kalman filter to estimate time-varying parameters.

3.1 Cointegration Analysis

Financial theory suggests that spot and futures prices are cointegrated. Accordingly, the cointegrating relationship between the natural logarithms of futures prices, f_t, and spot prices, s_t, at time $t = 1, \ldots, T$ can be formulated as:

$$f_t = \beta_0 + \beta_1 s_t + ec_t, \tag{1}$$

where ec_t denotes the estimated error correction term capturing the temporary deviations from the long-run equilibrium as a result of market frictions.

We employ the autoregressive distributed lag (ARDL) bounds test proposed by Pesaran and Shin (1999) and Pesaran et al. (2001) to test for cointegration.[3] The major advantage of this approach is that it does not require the underlying regressors to be purely $I(1)$, i.e., integrated of order one in the sense of Engle and Granger (1987). Instead, the bounds testing procedure is also applicable when it is not known with certainty whether each of the regressors is $I(0)$, $I(1)$ or fractionally integrated. In contrast to other cointegration approaches, this method is therefore not subject to the problems associated with pre-testing for unit roots. As shown by Cavanagh et al. (1995) and Elliott (1998), common unit root tests have low statistical power when the roots are near but not exactly one. Moreover, Wang and Tomek (2007) demonstrate that the outcome of unit root tests applied to commodity

[3] As a robustness check we also conduct the Engle and Granger (1987) procedure and the Johansen (1991, 1995) trace test in our cointegration analysis. The results support the findings from the ARDL bounds test.

spot prices is often sensitive to the specification of the test equation.[4]

To implement the bounds testing technique, we estimate the following unrestricted ECM:

$$\Delta s_t = \gamma_0 + \pi_1 s_{t-1} + \pi_2 f_{t-1} + \sum_{i=1}^{p} \varphi_{1,i} \Delta s_{t-i} + \sum_{j=1}^{q} \varphi_{2,j} \Delta f_{t-j} + \varepsilon_t, \tag{2}$$

where Δ denotes the difference operator, such that returns of spot and futures prices are calculated as $\Delta s_t = ln(S_t) - ln(S_{t-1})$ and $\Delta f_t = ln(F_t) - ln(F_{t-1})$, respectively. The number of lagged differences of spot prices is denoted by p, while q is the number of lagged differences of futures prices. γ_0 is the unrestricted intercept and ε_t are white noise errors.

The bounds testing procedure involves the estimation of Eq. (2) by OLS and the performance of two separate tests on the estimated coefficients. First, an F-test examining the null hypothesis that $\pi_1 = \pi_2 = 0$ (no cointegration) against the alternative that $\pi_1 \neq 0$ and $\pi_2 \neq 0$ (cointegration). And second, a t-test examining the null hypothesis that $\pi_1 = 0$ (no cointegration) against the alternative that $\pi_1 \neq 0$ (cointegration).

As shown by Pesaran et al. (2001), the distributions of the F- and t-statistics under the null hypotheses are non-standard as they depend on the order of integration of the two underlying time series. To overcome this problem, the authors provide two sets of critical values: a lower value assuming that all regressors are $I(0)$, and an upper value assuming that all regressors are $I(1)$. Consequently, the null hypothesis of no cointegration is accepted if the respective sample test statistic is below the corresponding lower critical value. By contrast, the null hypothesis is rejected if the test statistic exceeds the relevant upper critical value. In cases where the test statistic is in-between the two bounds, the results are inconclusive.

3.2 Vector Error Correction Model and Price Discovery Measures

If f_t and s_t are cointegrated, the two time series have a VECM representation that allows for an investigation of lead-lag relationships (Engle and Granger 1987; Johansen 1991). For this purpose, we estimate the following bivariate VECM:

[4]We observe this feature in our data which is reflected by the fact that the ADF and KPSS test results (not reported) are sensitive to both the inclusion of deterministic trends and changes in the lag structure.

$$\Delta s_t = \mu_{s,0} + \alpha_s ec_{t-1} + \sum_{i=1}^{p} \delta_{ss,i} \Delta s_{t-i} + \sum_{j=1}^{q} \delta_{sf,j} \Delta f_{t-j} + \varepsilon_{s,t} \tag{3a}$$

$$\Delta f_t = \mu_{f,0} + \alpha_f ec_{t-1} + \sum_{i=1}^{p} \delta_{fs,i} \Delta s_{t-i} + \sum_{j=1}^{q} \delta_{ff,j} \Delta f_{t-j} + \varepsilon_{f,t}, \tag{3b}$$

where $\mu_{s,0}$ and $\mu_{f,0}$ denote intercepts. $\varepsilon_{s,t}$ and $\varepsilon_{f,t}$ are error terms assumed to be serially uncorrelated with zero mean and covariance matrix $\mathbf{\Omega}$. The error correction term ec_{t-1} corresponds to the lagged residual from the cointegrating equation in (1), such that $ec_{t-1} = f_{t-1} - \beta_0 - \beta_1 s_{t-1}$.[5] The error correction coefficients, α_s and α_f, indicate the speed of adjustment towards the long-run equilibrium in response to a short-run deviation of the system. A statistically significant error correction coefficient in one of the two equations suggests that this market reacts to any disequilibrium between spot and futures prices. This is equivalent to saying that this market adjusts to price movements originating in the other market (Garbade and Silber 1983). For example, if the estimates return a statistically significant error correction coefficient for the spot market, α_s, but an insignificant coefficient for the futures market, α_f, the results are supportive of a dominant role of the futures market in the price discovery process.

Finally, the short-run dynamics of the system are indicated by the coefficients $\delta_{ss,i}$, $\delta_{sf,j}$, $\delta_{fs,i}$ and $\delta_{ff,j}$, where p and q denote the number of lags of first-differenced spot and futures prices, respectively.[6] We use the AIC, the SBC, and an LM-test for residual autocorrelation to determine the optimal number of lags.

To quantify the relative contribution of each market to price discovery, we calculate three price discovery measures. The first measure is based on the concept of common factor weights as proposed by Schwarz and Szakmary (1994) and Gonzalo and Granger (1995). This approach uses the relative magnitude of the adjustment coefficients in the VECM to assess each market's contribution to price discovery. Accordingly, we compute the common

[5] As a robustness check we also estimate our VECM with a restricted cointegrating vector, $\beta = (1, -1)'$. The results are similar to those reported in this paper.

[6] For the sake of clarity, we abstain from showing the estimates of the short-run parameters. The empirical results, however, are consistent with the reported findings and are available upon request.

factor weights of the futures (θ_f) and the spot market (θ_s) as follows:

$$\theta_f = \frac{|\alpha_s|}{|\alpha_s| + |\alpha_f|} \quad \text{and} \quad \theta_s = 1 - \theta_f = \frac{|\alpha_f|}{|\alpha_s| + |\alpha_f|}. \tag{4}$$

Since the denominator represents the total adjustment of both markets to any difference between spot and futures prices, the common factor weights measure the relative portion of total adjustment. The values of the common factor weights are restricted to the interval between zero and one. Their interpretation is straightforward: if $\theta_f = 1$, price discovery occurs entirely in the futures market, as the adjustment reaction falls completely on the spot market. Conversely, if $\theta_f = 0$, price discovery solely takes place in the spot market, as the adjustment reaction falls completely on the futures market. If both markets contribute equally to the price discovery process, this would result in a value of 0.5 for each common factor weight.

The second measure we estimate is the so-called information share developed by Hasbrouck (1995). According to this concept, a market's relative contribution to price discovery is defined as the proportion of the variance of the common efficient equilibrium price that can be attributed to this particular market. By denoting $\boldsymbol{\Psi\Omega\Psi'}$ as the variance of the common efficient equilibrium price, the information share of market j can be expressed as:

$$IS_j = \frac{([\boldsymbol{\Psi F}]_j)^2}{\boldsymbol{\Psi\Omega\Psi'}}, \qquad j = 1, 2, \tag{5}$$

where $\boldsymbol{F}$ is the Cholesky factorization of the estimated VECM variance-covariance matrix $\boldsymbol{\Omega}$ (i.e., the lower triangular (2 x 2) matrix such that $\boldsymbol{\Omega} = \boldsymbol{FF'}$) and $\boldsymbol{\Psi}$ represents the long-run impact matrix of dimension (1 x 2). By construction, $IS_1 + IS_2 = 1$.[7] Similar to the interpretation of the common factor weights, price discovery occurs predominantly in the market for which the information share exceeds the value of 0.5.

Our third measure of price discovery is the modified information share as proposed by Lien and Shrestha (2009). The advantage to the traditional information share is its independence

[7] The calculation of the information share measure is discussed in detail in Appendix A.

from the particular ordering of the time series in the VECM. The modified information share is given by:

$$MIS_j = \frac{([\boldsymbol{F}^*\boldsymbol{\Psi}]_j)^2}{\boldsymbol{\Psi}\boldsymbol{\Omega}\boldsymbol{\Psi}'}, \quad j = 1, 2, \tag{6}$$

where the matrix $\boldsymbol{F}^*$ is obtained from an eigendecomposition of $\boldsymbol{\Omega}$ (i.e., $\boldsymbol{\Omega} = \boldsymbol{F}^*(\boldsymbol{F}^*)'$). The interpretation is similar to the information share by Hasbrouk (1995): price discovery is dominated by the market for which the modified information share exceeds the critical value of 0.5.

3.3 Time-Variation in Price Discovery

The above outlined price discovery measures are commonly used in the literature. However, they only indicate where price discovery occurs *on average* and are not able to detect structural changes in the price discovery process. Since we focus on two thinly traded markets where participants cannot buy or sell futures contracts frequently, price movements are prone to changes that are not reflected by economic fundamentals (Black 1976). To overcome this drawback, we apply the Kalman filter technique to obtain time-varying parameters. We abstain from conducting rolling and/or recursive regressions due to the assumption of constant parameters in OLS estimations and due to the problem associated with defining optimal window lengths.

The state-space form of the VECM of Eqs. (3a) and (3b) reads:

$$\boldsymbol{y}_t = \boldsymbol{Z}_t\boldsymbol{\xi}_t + \boldsymbol{\epsilon}_t, \quad \boldsymbol{\epsilon}_t \sim N(\boldsymbol{0}, \boldsymbol{R}) \tag{7a}$$

$$\boldsymbol{\xi}_t = \boldsymbol{F}\boldsymbol{\xi}_{t-1} + \boldsymbol{\eta}_t, \quad \boldsymbol{\eta}_t \sim N(\boldsymbol{0}, \boldsymbol{Q}), \tag{7b}$$

where Eq. (7a) represents the measurement equation and Eq. (7b) the transition equation. Time-varying parameters are assumed to evolve according to a pure random walk and are represented by the vector $\boldsymbol{\xi}_t$. $\boldsymbol{F}$ is an identity matrix. The multivariate normally distributed error terms $\boldsymbol{\epsilon}_t$ and $\boldsymbol{\eta}_t$ are serially uncorrelated with zero mean and diagonal covariance matrices $\boldsymbol{R}$ and $\boldsymbol{Q}$, respectively. Appendix B contains the state-space model in matrix notation.

We first estimate the diagonals of $\boldsymbol{R}$ and $\boldsymbol{Q}$ via maximum likelihood and filter in the second step the parameters of the state variables in $\boldsymbol{\xi}_t$. The Kalman filter is a recursive algorithm for computing optimal estimates of the state variables for each point t, conditional on the information set available up to time t (Durbin and Koopman 2001; Kim and Nelson 1999). After having filtered the optimal states, we calculate time-varying common factor weights, similar to Eq. (4):

$$\theta_{f,t} = \frac{|\alpha_{s,t}|}{|\alpha_{s,t}| + |\alpha_{f,t}|} \quad \text{and} \quad \theta_{s,t} = 1 - \theta_{f,t} = \frac{|\alpha_{f,t}|}{|\alpha_{s,t}| + |\alpha_{f,t}|}. \tag{8}$$

4 Data Description

Our analysis is based on weekly data for spot and futures prices of two agricultural commodities, namely hog and piglet. The sample starts on July 20th, 2009, which corresponds to the introduction date of the two futures contracts at the Eurex in Frankfurt. The sample ends on April 25th, 2014. The size of a single futures contract is specified at 8,000 kilogram of hog slaughter weight and 100 piglets, respectively. Prices are quoted in Euro per kilogram of hog and Euro per piglet. Futures contracts expire on a monthly basis, implying simultaneous trading of several contracts (one per maturity month) for each commodity. The settlement of open contracts at expiration takes place in cash, payable on the first business day after the final settlement day. Our data source for all futures prices is Thomson Reuters Datastream. We use price observations from the most nearby contract, as this is typically the most actively traded one. To obtain a continuous time series, we roll over to the next nearby contract on the first day of the maturity month.

The underlying of each futures contract is a weekly spot price index representing the weighted average of spot prices across different cash markets. For hog, the index is calculated as the weighted average of prices for slaughter pigs in four Central European countries, namely Germany, the Netherlands, Belgium and Austria. German spot prices account for the largest weight (66.7 percent) relative to the other three countries which equally enter with a weight of 11.1 percent. With respect to Germany, two price quotations are incorporated in the calculation of the index, both of which account for 33.3 percent. The first series is the consolidated price ("Vereinigungspreis") determined every Friday by the German producer group for cattle and meat (Vereinigung der Erzeugergemeinschaften für Vieh und Fleisch, VEZG). As from March 31, 2014, the consolidated price's weekly publication date changed to Wednesday. The second series for Germany consists of a public price fixing, carried out every Wednesday by the German Federal Institute for Food and Agriculture. The spot prices from the Netherlands are the weekly average of prices from the hog market in Utrecht (Varkensbeurs). Belgian prices are from the Belgian market leader in pig production, Van Danis, and are determined every Wednesday. Finally, the spot price quotation from Austria is taken from the VLV-Notierung, which is an Austrian producer group for agricultural products. Until March 28, 2014, Austrian reference prices were determined every week on Friday, but the publication date was changed to Wednesday afterwards.

The Eurex piglet index is the result of aggregating local spot prices from the four most important trading zones in Germany. The spot prices from these four German regions (Schwäbisch Gmünd, Bavaria, North-Rhine Westphalia and Lower Saxony) are weekly average prices (Monday-to-Friday) and are published every Monday of the following week. Each price series accounts for a weight of 25 percent within the piglet index. The data source for all spot prices is the Eurex website.

To match the data frequency of futures prices with that of spot prices, we calculate five-day-averages (Monday-to-Friday) of futures price observations for each calendar week of the sample period. Figure 1 plots the price observations for each commodity in the period from July 2009 until April 2014. The graphs show a strong co-movement between both spot and futures prices, indicating a common stochastic trend. The movement of the piglet prices, however, behaves more inert than that of the hog prices which may be caused by the lower trading volume in this market.

[Figure 1 about here]

Table 1 contains descriptive statistics of our data. The means of the returns are positive, but close to zero. The summary statistics on maxima, minima and standard deviations reveal that piglet prices fluctuate more heavily than hog prices during the sample period.

[Table 1 about here]

Table 2 presents a summary of trading statistics for the two futures contracts in each year. Both trading volume and open interest reflect the low trading activity in both markets since their introduction in 2009. Depending on the reference year, average trading volume in the hog (piglet) futures market ranged between 11 (2) and 43 (11) contracts per week. Hence, average trading activity was lower in the piglet futures market compared to the hog market. This feature allows us to relate the empirical results to differences in trading activity between these two markets.

[Table 2 about here]

5 Empirical Results

5.1 Cointegration Analysis

The results of the ARDL bounds test are presented in Table 3. Based on different lag selection criteria, we choose an ARDL (2, 1) model in the case of hog, and an ARDL (2, 3) model in the case of piglet. The adequacy of our model is confirmed by high p-values of the LM test. Yet results are similar with other lag specifications. The estimates reported in Table 3 show that the F-values exceed the 1 percent upper critical values in all specifications, leading us to conclude that spot and futures prices are cointegrated. In line with these findings are the results of a t-test on the lagged dependent variable in the ARDL model, as the null of no cointegration is rejected at the 1 percent level in all cases.

In sum, our cointegration analysis provides robust evidence for a long-run equilibrium relationship between futures and spot prices.[8] While our findings are in line with the majority of studies testing for cointegration between futures and spot prices in actively traded markets (e.g., Brockman and Tse 1995; Yang et al. 2001), they contrast the mixed evidence on cointegration with regards to thinly traded futures markets (e.g., Fortenberry and Zapata 1997; Mattos and Garcia 2004).

[Table 3 about here]

5.2 Vector Error Correction Model and Price Discovery Measures

The presence of cointegration enables the analysis of price discovery by estimating bivariate VECMs. Panel A of Table 4 displays the time-invariant estimates for the two error-correction coefficients of each commodity over the entire sample. For both commodities, the adjustment coefficients for the spot market, α_s, are highly significant, whereas the adjustment coefficients for the futures market, α_f, are not statistically different from zero at any conventional level of significance. In addition, the two spot market coefficients have the expected positive sign, implying that when futures prices are above (below) their equilibrium value, spot prices adjust upward (downward) to correct the disequilibrium. This may be interpreted as evidence for a dominating role of the futures market in the price discovery process in that spot prices follow the price movements originating in the futures market but not vice versa.

In the next step, we quantify the relative contribution of each market to price discovery in the framework of our constant-parameter-model. The estimates of the three price discovery

[8] The results of the Engle and Granger (1987) methodology and the Johansen (1991, 1995) trace test support these findings, since both tests indicate cointegration at the 1 percent level for the two commodities. The results are available upon request.

measures are displayed in Panel B of Table 4. According to the common factor weights, the *average* contribution of the Eurex futures market to price discovery during the sample period is 85 percent for hog and 77 percent for piglet. The relative contribution of both futures markets turns out to be larger when the price discovery performance is measured by means of the information share metric, with values corresponding to 0.98 and 0.93 for the hog and piglet futures market, respectively. The modified information share suggests that between 64 and 74 percent of new information is first incorporated into futures prices and then transmitted to the spot market.

[Table 4 about here]

Irrespective of the price discovery measure, our findings from the constant-parameter-model thus robustly identify the Eurex futures markets as the trading venues where the majority of new information is processed. Moreover, a comparison of the results for both markets shows a stronger price discovery performance of the more liquid hog contract. The quantitative magnitudes of our results are similar to previous findings on price discovery in agricultural commodities markets (e.g., Brockman and Tse 1995).

5.3 Time-Variation in Price Discovery

Figure 2 displays the time-varying error correction coefficients and common factor weights based on the state-space model. The first column shows the parameters of the hog market and the second column represents the parameters of the piglet market. $\alpha_{s,t}$ and $\alpha_{f,t}$ denote the time-varying error correction coefficients of the spot and futures market, respectively. The shaded areas are the corresponding 95% confidence intervals. The parameters $\theta_{f,t}$ are the futures market's time-varying common factor weights, calculated by Eq. (8).

[Figure 2 about here]

The spot market's adjustment coefficient for hog is in line with the results of the time-invariant approach: it is statistically significant during the entire sample without any pattern of time-variation. This implies that the spot market constantly adjusts to deviations from the long-run equilibrium. The error-correction term for the futures market also corresponds to the results of the time-invariant model as the parameter is not statistically significant during the entire sample. The common factor weight ($\theta_{f,t}$) of the hog market also reflects the results of the constant approach as the value of $\theta_{f,t}$ is always above 0.5. This indicates that new information is first incorporated in the futures market and then transferred to the spot market.

Piglet's spot market adjustment coefficient greatly differs from the constant results since the parameter is only significant during three short periods, accompanied by heavy fluctuations. Moreover, the amplitude of the coefficient is much higher than that of the hog market. The common factor weight for piglet mirrors the strong fluctuations in $\alpha_{s,t}$, since the value of $\theta_{f,t}$ falls below 0.5 at various times. This shows strong variations in the price discovery process. These results contrast the findings from the time-invariant VECM and underscore the relevance of accounting for time-varying parameters, especially when investigating thinly traded futures markets.

We derive two conclusions from the results. First, the trading volume threshold which facilitates efficient price discovery lies in the range between the trading volume of the piglet market and the trading volume of the hog market. This argument is based on the evidence that price discovery for hog prices constantly occurs in the futures market, while price discovery for piglet prices alternates between both markets. Considering the fact that the hog market is also thinly traded, this trading volume threshold appears to be very low. Second, neglecting time variations in the analysis of price discovery can lead to misleading findings, especially when the results are based on data from thinly traded markets.

6 Conclusions

The question how much trading activity is needed to facilitate efficient price discovery in commodity futures markets is still unanswered in the literature. This paper therefore examines the price dynamics of two thinly traded agricultural futures contracts traded at the European Exchange in Frankfurt. We determine the relative contribution of each futures market to the price discovery process by estimating price discovery measures that are frequently applied in the literature. In addition, we account for time variation in the parameters.

The time-invariant price discovery measures indicate that both futures markets are dominant in the price discovery process. The results of the time-varying VECM, however, reveal a different pattern: the common factor weights for the less liquid piglet market fluctuate much more than those of the hog market, and fall below the critical value of 0.5 at various times. Since the hog market is also thinly traded, we argue that a relatively low level of trading volume is sufficient to facilitate efficient price discovery. This threshold has to be above the trading volume of the piglet market but below or equal to that of the hog market.

These results have two implications. First, market participants can rely on price signals from derivative exchanges, even if the futures market is thinly traded and barely used for hedging purposes. Trading volume therefore seems to be more relevant for attracting further hedgers and financial investors rather than to facilitate efficient price discovery. Second, the results may be of interest to regulators who doubt the benefits associated with the introduction of new futures contracts.

There is no doubt that more research has to be done to verify our argument of a low trading volume threshold for efficient price discovery. On the one hand, future research should investigate financial markets with a higher frequency of spot trading. On the other hand, empirical studies should try to increase the data frequency. The econometric approaches, however, should be based on constant and time-varying models since our findings imply that thinly traded futures markets are subject to strong fluctuations in the price discovery process which are not revealed within time-invariant econometric frameworks.

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Table 1: Descriptive Statistics

	Mean	Max.	Min.	Std. Dev.	Skewness	Kurtosis	Jarque-Bera
Δs_{hog}	0.0001	0.0832	-0.1036	0.0219	-0.2552	6.3019	115.3529***
Δf_{hog}	0.0003	0.0718	-0.0740	0.0215	-0.3345	4.5910	30.7817***
Δs_{piglet}	0.0007	0.1784	-0.2474	0.0316	-1.2889	22.3313	3882.6600***
Δf_{piglet}	0.0007	0.1081	-0.1117	0.0311	-0.3699	4.7762	37.7948***

The table shows descriptive statistics for the hog and piglet returns. Returns are calculated as first differences of log prices. *, **, *** denote statistical significance at the 10, 5, and 1 percent level, respectively.

Table 2: Trading Statistics of First Nearby Contract

	Panel A: Average Weekly Trading Volume					
	2009	2010	2011	2012	2013	2014
f_{hog}	40	43	39	35	17	11
f_{piglet}	8	11	5	6	3	2

	Panel B: Average Weekly Open Interest					
	2009	2010	2011	2012	2013	2014
f_{hog}	268	242	183	174	145	72
f_{piglet}	96	95	47	57	35	15

The data is taken from Thomson Reuters Datastream.

Table 3: Bounds Testing Procedure

	Deterministic components	ARDL (p^*, q^*) structure	F-statistic	t-statistic	p-value LM test
Hog	constant	(2, 1)	15.08***	-5.49***	0.87
	constant, trend	(2, 1)	15.06***	-5.47***	0.86
Piglet	constant	(2, 3)	14.25***	-5.34***	0.80
	constant, trend	(2, 3)	14.02***	-5.26***	0.79

The table shows empirical results for the bounds testing procedure. The ARDL-ECM in Eq. (2) is either estimated with a constant only or with a constant and a linear trend term. The optimal lag structure (p^*, q^*) is determined by a combination of the AIC, the SBC and LM tests on serial correlation in the residuals. F- and t-statistics refer to H_0: $\pi_1 = \pi_2 = 0$ and H_0: $\pi_1 = 0$ in Eq. (2), respectively. Cointegration is found if the estimated F- and t-statistics exceed the respective upper critical values reported in Pesaran and Shin (2001). *, **, and *** denote statistical significance at the 10, 5, and 1 percent level, respectively.

Table 4: Constant Estimation Approach

	Panel A: VECM Adjustment Coefficients	
	α_s	α_f
Hog	0.178***	0.034
Piglet	0.247***	0.074

	Panel B: Price Discovery Measures					
	CFW		IS		MIS	
	Futures	Spot	Futures	Spot	Futures	Spot
Hog	0.85	0.15	0.98	0.02	0.74	0.26
Piglet	0.77	0.23	0.93	0.07	0.64	0.36

Panel A shows the estimates of the adjustment coefficients obtained from estimating the bivariate VECM in Eqs. (3a) and (3b) over the entire sample period (July 2009-April 2014). Coefficients of the spot and futures market are denoted by α_s and α_f, respectively. *, **, *** denote statistical significance at the 10, 5, and 1 percent level, respectively. Panel B reports the estimates of three price discovery measures calculated for the whole sample period. CFW stands for the common factor weights proposed by Schwarz and Szakmary (1994) and Gonzalo and Granger (1995). IS is the information share (mean of upper and lower bound) by Hasbrouck (1995), and MIS denotes the modified information share developed by Lien and Shrestha (2009).

Figure 1: Spot and Futures Prices (Weekly)

(a) Hog

(b) Piglet

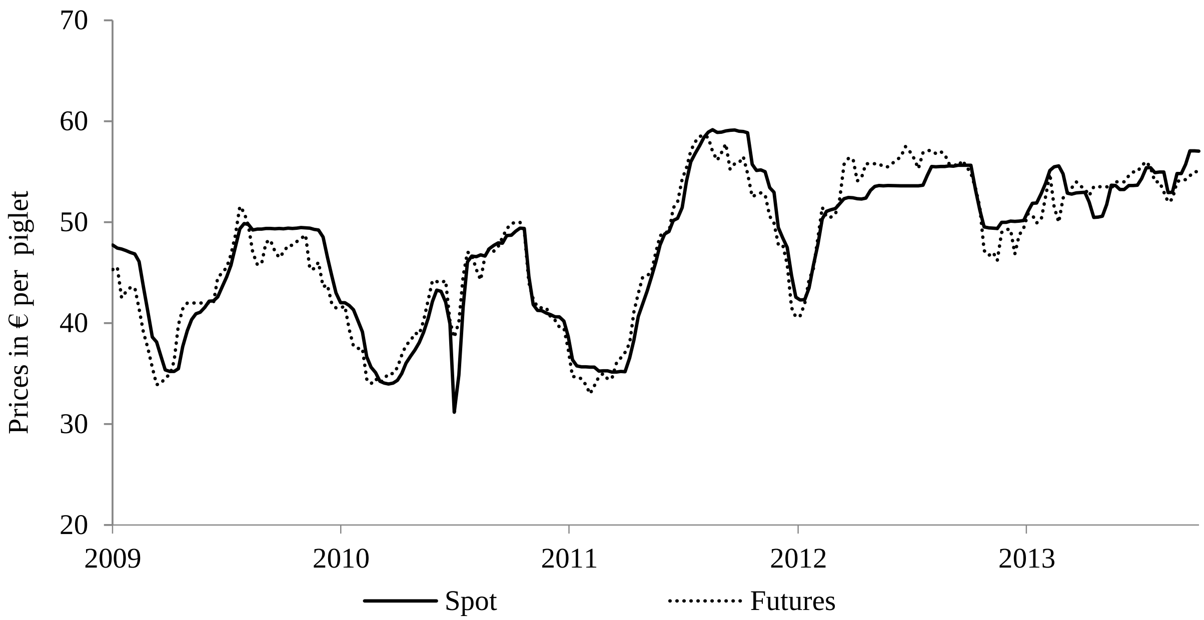
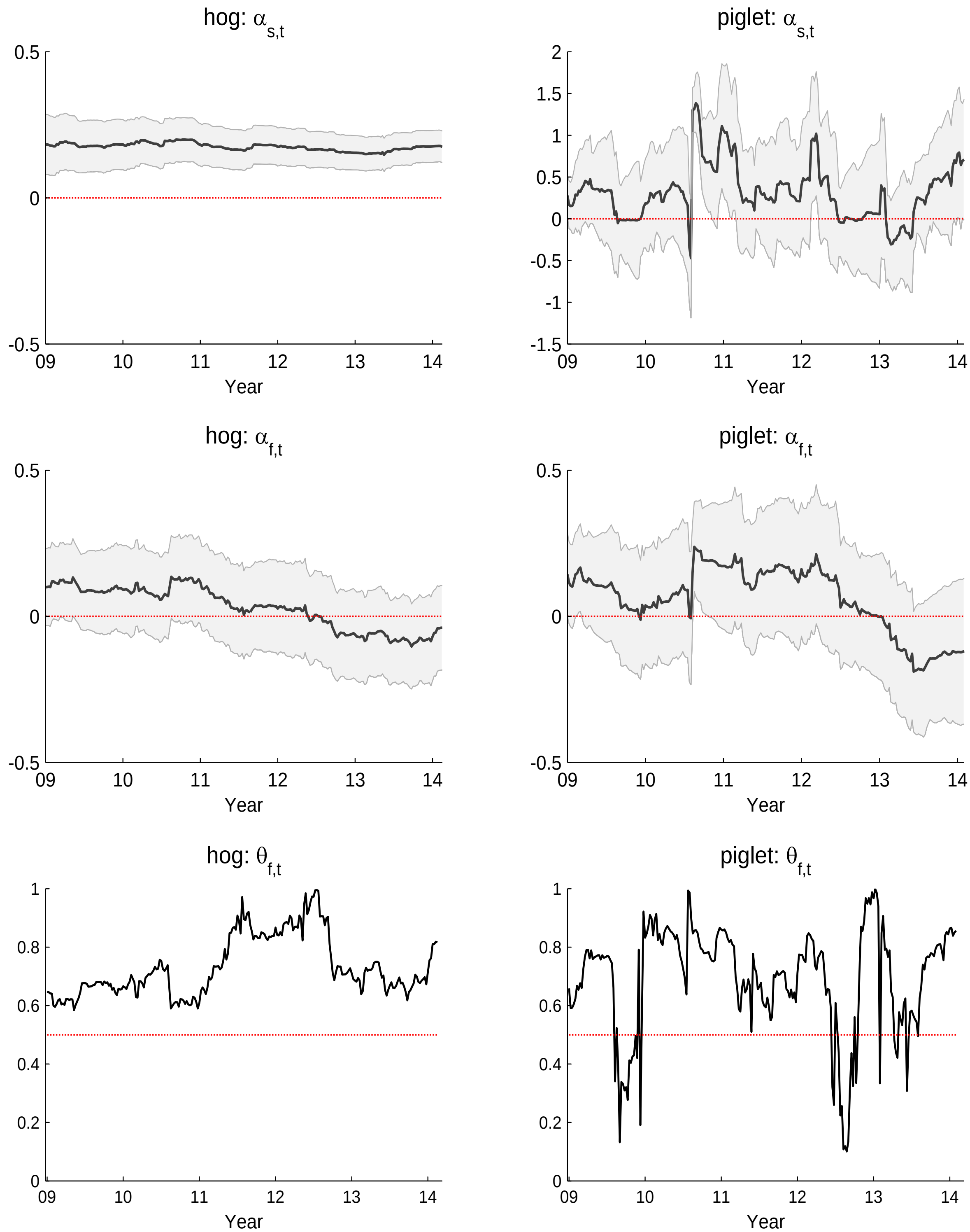


Figure 2: Time-Varying Parameters



The first column shows time-varying results for the hog market and the second column for the piglet market. $\alpha_{s,t}$ and $\alpha_{f,t}$ are the time-varying error correction coefficients for the spot and futures market, respectively. The bold lines show the point estimates and the shaded areas the 95% confidence intervals. $\theta_{f,t}$ represents the common factor weight (CFW) of the futures market, estimated by Eq. (8).

Appendix A: Information Share Measure

Hasbrouck (1995) shows that the VECM given in Eqs. (3a) and (3b) can be transformed into the following vector moving average (VMA) representation in levels:

$$\boldsymbol{p_t} = \boldsymbol{p_0} + \boldsymbol{\Psi(1)} \sum_{i=1}^{t} \boldsymbol{\varepsilon_i} + \boldsymbol{\Psi^*(L)\varepsilon_t}, \tag{9}$$

where $\boldsymbol{p_t} = (s_t, f_t)'$ is the two-dimensional price vector containing the series of spot and futures prices (denoted by s_t and f_t, respectively), $\boldsymbol{p_0}$ is the (2 x 1) vector of constants, $\boldsymbol{\varepsilon_t}$ the (2 x 1) error vector with covariance matrix $\boldsymbol{\Omega}$, and $\boldsymbol{\Psi^*(L)}$ is a matrix polynomial in the lag operator, L. The elements of the (2 x 2) moving average impact matrix, $\boldsymbol{\Psi(1)}$, correspond to the cumulative VMA coefficients. Moreover, the term $\boldsymbol{\Psi(1)} \sum_{i=1}^{t} \boldsymbol{\varepsilon_i}$ constitutes the long-run impact (i.e., the stochastic trend) of an innovation at time t on each of the prices. By construction, the rows of the impact matrix $\boldsymbol{\Psi(1)}$ are identical. By specifying $\boldsymbol{\Psi} = (\Psi_1, \Psi_2)$ as the common row vector in $\boldsymbol{\Psi(1)}$, Hasbrouck (1995) defines the stochastic trend component $\boldsymbol{\Psi\varepsilon_t}$ as the common efficient price between the two price series. He proposes to decompose the variance of this term, i.e., var$(\boldsymbol{\Psi\varepsilon_t}) = \boldsymbol{\Psi\Omega\Psi'}$, to attribute the proportion of variance in the common component to innovations in each of the two markets. Hence, if the errors of the VECM are uncorrelated, implying that $\boldsymbol{\Omega}$ is diagonal, then $\boldsymbol{\Psi\Omega\Psi'}$ will consist of two terms, each of which represents the contribution to the variance of the common efficient price from the corresponding market. Accordingly, the contribution of market j relative to the total variance is defined as market j's information share:

$$IS_j = \frac{\Psi_j^2 \sigma_j}{\boldsymbol{\Psi\Omega\Psi'}}, \quad j = 1, 2 \tag{10}$$

where Ψ_j is the j-th element of $\boldsymbol{\Psi}$ and σ_j is the j-th diagonal element of $\boldsymbol{\Omega}$.

Since $\boldsymbol{\Omega}$ will not be diagonal if the residuals are correlated, it is crucial to eliminate the residual correlation. For this purpose, Hasbrouck (1995) suggests to make use of the Cholesky factorization of $\boldsymbol{\Omega}$ such that $\boldsymbol{\Omega = FF'}$, where $\boldsymbol{F}$ is a lower triangular matrix. Substituting this expression into Eq. (10) yields an information share of market j that is given by:

$$IS_j = \frac{([\boldsymbol{\Psi F}]_j)^2}{\boldsymbol{\Psi \Omega \Psi'}}, \tag{11}$$

where $[\boldsymbol{\Psi F}]_j$ is the *j-th* element of the row matrix $\boldsymbol{\Psi F}$.

Finally, Baillie et al. (2002) prove that the information share measure can be estimated directly from the VECM, thereby facilitating the estimation process. They show that the impact matrix $\boldsymbol{\Psi(1)}$ depends on the orthogonals of the error correction coefficients, $\boldsymbol{\alpha}_\perp$, and the cointegrating vector, $\boldsymbol{\beta}_\perp$. Hence, for $\boldsymbol{\beta} = (1, -1)'$ and $\boldsymbol{\beta}_\perp = (1, 1)'$, they derive the following expression for $\boldsymbol{\Psi(1)}$:

$$\boldsymbol{\Psi(1)} = \boldsymbol{\beta}_\perp \pi \boldsymbol{\alpha}'_\perp = \begin{pmatrix} \boldsymbol{\Psi} \\ \boldsymbol{\Psi} \end{pmatrix} = \begin{pmatrix} \Psi_1 & \Psi_2 \\ \Psi_1 & \Psi_2 \end{pmatrix} = \pi \begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_1 & \gamma_2 \end{pmatrix}, \tag{12}$$

where γ_1 and γ_2 are the orthogonals to the vector of error correction coefficients ($\boldsymbol{\alpha}_\perp$), and π represents a scalar.

Substituting the expression for $\boldsymbol{\Psi}$ in Eq. (12) into Eq. (11) yields a formulation of the information share that depends only on the adjustment coefficients and the variance-covariance matrix $\boldsymbol{\Omega}$:

$$IS_j = \frac{([\boldsymbol{\gamma F}]_j)^2}{\boldsymbol{\gamma \Omega \gamma'}}, \tag{13}$$

where $\boldsymbol{\gamma} = (\gamma_1, \gamma_2)' = \boldsymbol{\alpha}_\perp$ is the common row vector containing the orthogonals to the error correction coefficients and $[\boldsymbol{\gamma F}]_j$ is the *j-th* element of the row of the matrix $\boldsymbol{\gamma F}$.

Since the factorization imposes a greater weight on the first price series in the VECM, the computed value of each market's information share depends on the ordering of the variables in the estimation process. We therefore report the mean of upper and lower bounds for each information share.

Appendix B: State-Space Model

Measurement Equation:

$$\begin{bmatrix} \Delta s_t \\ \Delta f_t \end{bmatrix} = \begin{bmatrix} 1 & ec_{t-1} & \Delta s_{t-1} & \Delta f_{t-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & ec_{t-1} & \Delta s_{t-1} & \Delta f_{t-1} \end{bmatrix} \begin{bmatrix} \mu_{s0,t} \\ \alpha_{s,t} \\ \delta_{ss,t} \\ \delta_{sf,t} \\ \mu_{f0,t} \\ \alpha_{f,t} \\ \delta_{fs,t} \\ \delta_{ff,t} \end{bmatrix} + \begin{bmatrix} \epsilon_{s,t} \\ \epsilon_{f,t} \end{bmatrix} \tag{14}$$

$$\boldsymbol{y}_t = \boldsymbol{Z}_t \boldsymbol{\xi}_t + \boldsymbol{\epsilon}_t, \boldsymbol{\epsilon}_t \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2_{\epsilon_s} & 0 \\ 0 & \sigma^2_{\epsilon_f} \end{bmatrix} \right)$$

Transition Equation:

$$\begin{bmatrix} \mu_{s0,t} \\ \alpha_{s,t} \\ \delta_{ss,t} \\ \delta_{sf,t} \\ \mu_{f0,t} \\ \alpha_{f,t} \\ \delta_{fs,t} \\ \delta_{ff,t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mu_{s0,t-1} \\ \alpha_{s,t-1} \\ \delta_{ss,t-1} \\ \delta_{sf,t-1} \\ \mu_{f0,t-1} \\ \alpha_{f,t-1} \\ \delta_{fs,t-1} \\ \delta_{ff,t-1} \end{bmatrix} + \begin{bmatrix} \eta_{s0,t} \\ \eta_{s,t} \\ \eta_{ss,t} \\ \eta_{sf,t} \\ \eta_{f0,t} \\ \eta_{f,t} \\ \eta_{fs,t} \\ \eta_{ff,t} \end{bmatrix} \tag{15}$$

$$\boldsymbol{\xi}_t = \boldsymbol{F} \boldsymbol{\xi}_{t-1} + \boldsymbol{\eta}_t \ , \boldsymbol{\eta}_t \sim N\left(\begin{bmatrix} 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2_{\eta_{s0}} & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \sigma^2_{\eta_{fs}} & \\ 0 & \cdots & & \sigma^2_{\eta_{ff}} \end{bmatrix} \right)$$