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Human Capital, Growth, and Asset Prices

Fabian Goessling [†]

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[†] Department of Economics, University of Münster, Germany

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Abstract

Human capital investment is one of the most important drivers of growth. In this paper, I enhance the endogenous growth model of Kung and Schmid (2015) by an educational choice decision of the household. The engine of growth in the extended model is a composite of firm-side R&D capital stock and a household-side human capital stock. As households are able to allocate their time to labor, leisure and educational activities, the model allows to simultaneously study labor and education choices. The parameters of the non-linear model can be estimated by advanced Bayesian methods, allowing to study the return on human capital conditional on the data.

Keywords: Asset Pricing, Endogenous Growth, Human Capital, Bayesian Estimation
JEL: E23, G12, I26, J24, O24

1. Introduction

While labor income, i.e. the return to human capital, constitutes a major share of total income for most people, the majority of modern general equilibrium asset pricing models does not explicitly account for the accumulation of human capital and the corresponding optimal decisions. This is especially surprising given the early pioneering work of Uzawa (1965), Romer (1986) and particularly Lucas (1988). Even though these authors primarily focus on macroeconomic implications and economic development, their work constitutes a starting point to jointly study human capital, its accumulation and its return in a production based setting. Still, subsequent models as e.g. Dang et al. (2012), Dejong and Ingram (2001) Malley and Woitek (2011) or Stokey (2012) follow a macroeconomic centered paradigm, which results in close to zero risk premia. A notable exception is Wei (2005), who is able to generate small equity premia using time-to-build frictions. Nevertheless, CRRA utility functions and a consumption process that behaves almost like a random walk severely limit the asset pricing capabilities.

In the present paper, I propose a human capital model, which does not suffer from such shortcomings. In order to meaningfully study returns on human capital, I suggest an extended version of the Kung and Schmid (2015) model of endogenous growth that can generate sizeable risk premia. This approach endogenously creates *long-run risk*, i.e. a

Email address: `Fabian.Goessling@uni-muenster.de` (Fabian Goessling)
[1]Department of Economics, Am Stadtgraben 9, University of Münster 48143 Münster, Germany

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small but persistent component in consumption growth that has previously been modeled by Bansal and Yaron (2004) or Croce (2014) as an exogenous process. Such a framework in combination with Epstein and Zin (1989) preferences and Jermann (1998) adjustment costs for physical capital accumulation allows to match several key asset pricing moments.

While preserving the general structure of the Kung and Schmid (2015) model, which has its roots in Romer (1990) and Comin and Gertler (2006), I add a parsimonious human capital accumulation rule. Human capital builds up as time is allocated to education and schooling activities. The stock of human capital represents the cognitive abilities and skills of the household and constitutes a non-perfect substitute to organisational R&D capital as in Stokey (2014). As a result, the model is driven by a *"twin engine of growth"* (Stokey (2012)), which is fuelled by firms investing in organisational R&D seeking higher productivity, and households investing in human capital seeking higher wages. Furthermore, I adopt the leisure-labor decision as studied by Donadelli and Grüning (2016).

A substantial methodological and empirical contribution of the present paper is the estimation of the production based asset pricing model using advanced Bayesian techniques, following Goessling (2018). The pertinent literature almost exclusively relies on calibration exercises where the model parameters are set to "reasonable" but arbitrary values. In contrast, the Bayesian estimation of the non-linear model allows to quantify the return on human capital, conditional on the data, and also provides posterior densities for all parameters of interest.

The remainder of the paper is as follows. Section 2 introduces the model and derives the optimality conditions. Section 3 describes the estimation procedure and the data. The estimation results are presented in section 4. Section 5 concludes.

2. Model

This section presents the human capital model. I derive the optimality conditions and summarize the economic interactions between the productive sector and households. Additionally, I introduce the asset prices which are of particular interest for the estimation in Section 3.

2.1. Production

2.1.1. Final Good Sector

The final consumption good is produced by a representative firm that uses physical capital K_t, labor hours L_t and a bundle

$$\mathcal{X}_t^{\frac{1}{\nu}} = \left(\int_{i=0}^{G_t} X_{i,t}^{\nu} di \right)^{\frac{1}{\nu}}$$

of intermediate goods $X_{i,t}$ with $i \in [0, G_t]$, according to the constant returns to scale production function

$$Y_t = \left(K_t^{\alpha} (\Omega_t L_t)^{1-\alpha} \right)^{1-\xi} \left(\mathcal{X}_t^{\frac{1}{\nu}} \right)^{\xi}.$$

The parameter α is the capital share, ξ is the intermediate good share and ν is the inverse markup.

In contrast to Kung and Schmid (2015) the measure of intermediate goods is a CES aggregate

$$G_t = \left(\omega N_t^{\frac{\chi-1}{\chi}} + (1-\omega) H_t^{\frac{\chi-1}{\chi}} \right)^{\frac{\chi}{\chi-1}}$$

as in Stokey (2014), where N_t is the organisational R&D capital and H_t is human capital provided by the households with weight parameter ω. Moreover, χ is the elasticity of substitution between organisational capital and human capital. For $\chi \to 1$ the function nests the Cobb-Douglas case, and $\chi = 0$ is the Leontief case. The labor augmenting technological progress Ω_t is an exogenous stochastic process,

$$\Omega_t = \exp(\epsilon_t), \;\; \epsilon_t \sim \mathcal{N}(\rho \epsilon_{t-1}, \sigma^2). \tag{1}$$

The final good producing firm maximizes shareholder value using the pricing kernel M_t of the household, i.e.

$$\max_{\{I_t, L_t, K_{t+1}, X_{i,t}\}_{t>0, i \in [0, G_t]}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} M_{t+1} D_{t+1} \right],$$

where dividends D_t are defined as output Y_t less capital investment I_t, wages $W_t L_t$ and the total cost of purchasing intermediate patented goods $X_{i,t}$ at a price $P_{i,t}$, i.e.

$$D_t = Y_t - I_t - W_t L_t - \int_{i=0}^{G_t} P_{i,t} X_{i,t} \, di.$$

The evolution of the capital stock K_t is

$$K_{t+1} = (1 - \delta^K) K_t + \Phi \left(\frac{I_t}{K_t} \right) K_t, \tag{2}$$

where $\Phi(\cdot)$ is the convex adjustment cost function of Jermann (1998)

$$\Phi(x) = \frac{b_1}{1-\kappa} x^{1-\kappa} + b_2.$$

Thus fluctuations from steady state investment become increasingly costly, preventing excessive smoothing.[2] The optimal investment decision yields the usual Euler equation

$$\tilde{q}_t = \frac{1}{\Phi'(\frac{I_t}{K_t})}, \tag{3}$$

$$\tilde{q}_t = \mathbb{E}_t \left[M_{t+1} \left(\alpha(1-\xi)\frac{Y_{t+1}}{K_{t+1}} - \frac{I_{t+1}}{K_{t+1}} + \tilde{q}_{t+1} \left(1 - \delta^K + \Phi\left(\frac{I_{t+1}}{K_{t+1}}\right)\right)\right)\right]. \tag{4}$$

Furthermore, optimality yields the demand for labor hours L_t as

$$W_t = \frac{(1-\xi)(1-\alpha)Y_t}{L_t}$$

and the demand for intermediate goods $X_{i,t}$ as a function of the corresponding price $P_{i,t}$ as

$$X_{i,t}(P_{i,t}) = \left(\xi \frac{Y_t}{P_{i,t}}\right)^{\frac{1}{1-\nu}} \mathcal{X}_t^{\frac{1}{\nu-1}}.$$

2.1.2. Intermediate Sector

Differentiated intermediate goods $X_{i,t}$ are provided by monopolistic competitive firms, which transform consumption goods into intermediate goods at marginal costs of unity. Thus, the optimal price $P_{i,t}$ is the solution of the profit maximization problem

$$\Pi_{i,t} = \max_{P_{i,t}} P_{i,t} X_{i,t}(P_{i,t}) - X_{i,t}(P_{i,t}). \tag{5}$$

The inverse demand curve of the final good producer, i.e.

$$P_{i,t} = \xi Y_t \mathcal{X}_t^{-1} X_{i,t}^{\nu-1},$$

is taken as given by the intermediate sector which implies a symmetric solution to (5) as

$$P_{i,t} \equiv P_t = \frac{1}{\nu},$$
$$X_{i,t} \equiv X_t.$$

Substituting the optimal price in the inverse demand function yields

$$X_t = {K_t}^{\alpha} {\Omega_t}^{1-\alpha} {L_t}^{1-\alpha} {G_t}^{\frac{-\xi+\nu}{\nu(\xi-1)}} (\xi\nu)^{-\frac{1}{\xi-1}}, \tag{6}$$

[2]Following common practice, I set the parameters b_1 and b_2 such that in the deterministic steady state (denoted by variables without time index) the adjustment costs equal zero. That is, the conditions $\Phi(I/K) = I$ and $\Phi'(I/K) = 0$ imply $b_1 = (g-1+\delta^K)^{\kappa}$ and $b_2 = (g-1+\delta^K)(1-1/(1-\kappa))$ where g is the deterministic steady state growth rate of the model.

hence the profits are

$$\Pi_{i,t} \equiv \Pi_t = \left(\frac{1}{\nu} - 1\right) X_t. \tag{7}$$

Given the profit definition, the value of providing the differentiated good X_t is

$$V_t = \Pi_t + (1 - \phi_t)\mathbb{E}_t[M_{t+1}V_{t+1}], \tag{8}$$

where ϕ_t is the probability that the intermediate good i becomes obsolete. Note that in contrast to Kung and Schmid (2015), the probability ϕ_t is not determined exclusively by the depreciation rate of organisational R&D, but also includes the depreciation rate of human capital,

$$\phi_t = 1 - \frac{\hat{G}_{t+1}}{G_t},$$

where $\hat{G}_{t+1}$ is the measure of innovations under the assumption of no investment in H_t and N_t in period t.

Balanced growth requires the restriction

$$\nu = \frac{\xi}{\alpha\,\xi - \alpha + 1}, \tag{9}$$

such that final output can be written as

$$Y_t = A_t K_t^{\alpha} (G_t L_t)^{1-\alpha},$$

where the productivity level is defined as

$$A_t = \Omega_t{}^{1-\alpha} \xi^{-\frac{\xi}{\xi-1}} \nu^{-\frac{\xi}{\xi-1}}. \tag{10}$$

Thus, in equilibrium, the bundle G_t of organisational and human capital turns endogenously out to be labor augmenting. This result mirrors the fact that R&D and human capital increase the efficiency of plain labor hours. Furthermore, sustained growth around a balanced growth path arises endogenously as an outcome of the agents' decisions since the production function is homogeneous of degree one in the accumulating factors K_t, N_t and H_t.

2.1.3. Innovation Sector

The innovation sector combines the organisational research capital of the productive sector with the human capital provided by households. Note that while human capital is embodied in the labor time provided to the productive sector, R&D capital has to be accumulated by investing I_t^N following the accumulation rule

$$N_{t+1} = \vartheta_t I_t^N + (1 - \delta^N) N_t, \tag{11}$$

where

$$\vartheta_t = \bar{\vartheta} \left(\frac{N_t}{I_t^N} \right)^{1-\eta^N} \tag{12}$$

is the productivity externality of Kung and Schmid (2015) and Comin and Gertler (2006).

Free entry in the intermediate patented good producing sector implies that any new idea will be adopted. Consequently, the innovative sector sells the right to produce the good i at a price which equals the value to patented good producers, i.e. V_t. Investing in organisational capital is advantageous as long as the expected discounted marginal revenue is larger than the marginal cost of investing (which equals unity), i.e.

$$1 = \mathbb{E}_t \left[M_{t+1} V_{t+1} \frac{\partial G_{t+1}}{\partial N_{t+1}} \frac{\partial N_{t+1}}{\partial I_t^N} \right]. \tag{13}$$

Note that in the limiting case $G_t = N_t$, condition (13) reduces to

$$\frac{1}{\vartheta_t} = \mathbb{E}_t[M_{t+1} V_{t+1}],$$

which is equation (20) in Kung and Schmid (2015).

2.2. Representative Household

2.2.1. Utility & Wages

The representative household has Epstein and Zin (1989) preferences,

$$U_t = \left[(1-\beta) u_t^{\frac{1-\gamma}{\theta}} + \beta \big(\mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \big)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}},$$

where β is a time-preference parameter and u_t the intratemporal utility. I adopt the notation of Caldara et al. (2012) and define the composite parameter

$$\theta = \frac{1-\gamma}{1-\frac{1}{\psi}},$$

which can be interpreted as a measure of the deviation to the CRRA-utility case ($\theta = 1$). In particular, γ is the risk aversion parameter and ψ the intertemporal elasticity of substitution. In contrast to Kung and Schmid (2015) or Donadelli and Grüning (2016), I extend the model by introducing an education choice. Intratemporal utility u_t is given as

$$u(C_t, E_t, L_t) = C_t \left(\bar{T} - L_t - E_t \right)^{\tau},$$

where C_t is consumption, L_t labor time and E_t the time invested in education, i.e. schooling time. Overall available time is denoted by $\bar{T}$ and the elasticity of non-leisure time is τ. The budget constraint is

$$C_t = D_t^a + W_t L_t,$$

thus consumption C_t is financed by aggregate dividends D_t^a and labor income $W_t L_t$. Solving the decision problem of the household yields the required compensation for labor time as

$$W_t = \frac{\tau C_t}{(\bar{T} - L_t - E_t)}. \tag{14}$$

2.2.2. Human Capital Investment

The evolution of the human capital stock is determined by

$$H_{t+1} = \Psi_t H_t E_t + (1 - \delta^H) H_t, \tag{15}$$

where E_t is the education time provided by the households and δ^H is the depreciation rate of human capital. Thus, the specification is similar to Dang et al. (2012) or Malley and Woitek (2011) with a minor difference: In the spirit of Comin and Gertler (2006) and Kung and Schmid (2015), Ψ_t is taken as given by the representative household, but evolves as

$$\Psi_t = \bar{\Psi} \frac{1}{E_t^{1-\eta^H}}, \tag{16}$$

where $\eta^H \in [0, 1]$ is the elasticity of new human capital with respect to time devoted to education.[3] As a consequence, $\frac{\partial \Psi_t}{\partial E_t} < 0$, i.e. the production technology of new human capital exhibits diminishing returns with respect to education.

Concerning the education choice, the decision to allocate time to schooling and training is made with a *believed* return in mind, i.e. a belief on $\tilde{W}'(H_t) = \frac{\partial W_t}{\partial H_t} > 0$ as there is no direct compensation. As an example, public knowledge might have it that the marginal wage with respect to human capital depends on the level of human capital (e.g. proxied by occupational groups). For parsimony, I adopt a simplified variant where the households take present marginal wages as a reference for future marginal wages, denoted as $\tilde{W}'(H_{t+1})$. Thus

$$\tilde{W}'(H_{t+1}) = \frac{\partial W_t}{\partial G_t} \frac{\partial G_t}{\partial H_t}.$$

Using the first order condition with respect to education

$$\frac{\partial u_t}{\partial E_t} = \lambda_t^2 \Psi_t H_t$$

[3]Note that the asymmetry between human capital accumulation (15) and R&D accumulation (11) is a technical necessity, since the time devoted to education E_t cannot grow in equilibrium. Consequently, education time E_t has to be scaled by a growing variable, e.g. the level of human capital H_t, for balanced growth to exist.

allows to relate the Lagrange multiplier of the budget constraint λ_t^1 and the Lagrange multiplier of the human capital equation λ_t^2 by

$$\lambda_t^1 W_t = \lambda_t^2 \Psi_t H_t.$$

Optimality with respect to H_{t+1} yields

$$\lambda_t^2 = \mathbb{E}_t \left[\lambda_{t+1}^1 \tilde{W}'(H_{t+1}) + \lambda_{t+1}^2 (\Psi_{t+1} E_{t+1} + (1 - \delta^H)) \right],$$

where substituting the Lagrange multipliers yields

$$\frac{W_t}{\Psi_t H_t} = \mathbb{E}_t \left[M_{t+1} \left(\tilde{W}'(H_{t+1}) + \frac{W_{t+1}}{\Psi_{t+1} H_{t+1}} (\Psi_{t+1} E_{t+1} + (1 - \delta^H)) \right) \right] \tag{17}$$

and the households pricing kernel is defined as

$$M_{t+1} = \beta \left(\frac{U_{t+1}^{1-\gamma}}{\mathbb{E}_t[U_{t+1}^{1-\gamma}]} \right)^{1-\frac{1}{\theta}} \left(\frac{u_{t+1}}{u_t} \right)^{\frac{1-\gamma}{\theta}-1} \left(\frac{\frac{\partial u_{t+1}}{\partial c_{t+1}}}{\frac{\partial u_t}{\partial c_t}} \right). \tag{18}$$

2.3. Aggregate Resource Constraint

Final output is consumed, invested into new capital stock, invested in new R&D capital, and used to buy intermediate goods, hence market clearing implies

$$Y_t = C_t + I_t + I_t^N + G_t X_t. \tag{19}$$

2.4. Returns and Asset Prices

The risk free rate is the inverse of the expected stochastic discount factor,

$$R_t^f = \frac{1}{\mathbb{E}_t[M_{t+1}]}.$$

Using the model specification, I study the asset pricing implications of three particular assets: stocks of the final good producing firm, a claim to aggregate market dividends, and a claim to human capital returns. Recalling that the dividend of the final good sector is

$$D_t^d = Y_t - I_t - W_t^L L_t - \frac{1}{\nu} G_t X_t,$$

the corresponding stock price and its (log) return are

$$V_{t+1}^d = D_t^d + \mathbb{E}_t[M_{t+1} V_{t+1}^d],$$
$$r_t^d = \log \left(\frac{V_t^d}{V_{t-1}^d - D_{t-1}^d} \right).$$

Correspondingly, given the aggregate market dividend

$$D_t^a = C_t - W_t L_t,$$

the price of the market claim and its (log) return are

$$V_t^a = D_t^a + \mathbb{E}_t[M_{t+1} V_{t+1}^a],$$
$$r_t^a = \log\left(\frac{V_t^a}{V_{t-1}^a - D_{t-1}^a}\right).$$

Furthermore, I model a hypothetical claim to human capital returns with dividends

$$D_t^H = W_t \tag{20}$$

leading to the prices V_t^H and (log) return r_t^H. Note that in contrast to consumption based models with an exogenous labor/leisure process as in Dittmar et al. (2014), an additional claim based on $W_t L_t$ provides little insights as the labor choice is not prone to additional sources of risk.

2.5. Steady State & Solution

To solve the model all growing variables must be detrended. I use the level of organisational capital N_t as deflator, such that e.g. stationarized consumption is defined as

$$\hat{C}_t = \frac{C_t}{N_t}.$$

Deterministic steady state variables are written without time indices. Thus, in steady state

$$\hat{C} \equiv \hat{C}_t \equiv \hat{C}_{t+1}.$$

Given the stationarized equilibrium conditions, the system of equations is reduced to a minimum and then solved numerically. In particular, I solve the model in log-variables, where log (detrended) consumption $\hat{c}_t$ is defined by

$$\hat{C}_t = \exp(\hat{c}_t).$$

Following Kung and Schmid (2015) the policy functions are computed using a second order perturbation around the deterministic steady state.

3. Estimation method and data

Production based asset pricing models are commonly calibrated rather than rigorously estimated. In the next subsection, I suggest a Bayesian approach where only few parameters need calibration, whereas the joint posterior density of all remaining parameters is properly estimated. Based on the posterior distribution, population moments are compared to their empirical counterparts. I also report Bayesian Impulse Response Functions (BIRF) and discuss their implications. Finally, I compute the distribution of human capital returns.

3.1. Estimation method

The non-linear model is estimated by a randomized Quasi Sequential Markov Chain Monte Carlo technique.[4] The estimation approach combines Gerber and Chopin (2015) and Deligiannidis et al. (2016). It makes use of two particle filters to approximate the likelihood and the posterior density.[5] Several tuning approaches, randomized quasi Monte Carlo numbers, and Markov Chain Monte Carlo moves make the technique flexible, easy to parallelize and clearly superior to standard MCMC variants.

The estimation algorithm generates samples from the joint posterior distribution of parameters $\boldsymbol{\theta}$ given data $\mathbf{y}_{1:T} = \{\mathbf{y}_1, \dots, \mathbf{y}_T\}$,

$$p(\boldsymbol{\theta}|\mathbf{y}_{1:T}) \propto p(\mathbf{y}_{1:T}|\boldsymbol{\theta})p(\boldsymbol{\theta}),$$

and thus allows to estimate any function of interest of $\boldsymbol{\theta}$.[6] The expression $p(\mathbf{y}_{1:T}|\boldsymbol{\theta})$ is the likelihood of the data $\mathbf{y}_{1:T}$ conditional on the parameters $\boldsymbol{\theta}$, and $p(\boldsymbol{\theta})$ is the density of the prior distribution. The subset of parameters that are estimated and their prior distributions are listed in Table 1.

Some model parameters are calibrated. In particular, I fix β and the mean growth rate g, such that the deterministic steady state of the risk-free rate is close to its empirical counterpart. Furthermore, I presuppose that a third of the available time is allocated to labor, and ten percent is allocated to education. Thus, the balanced growth restriction (9) and the restrictions on the steady states jointly determine the values $\bar{\varphi}^H$, $\bar{\varphi}^N$,ν and τ. The depreciation rate of human capital, δ^H, is set to 0.04 which is roughly in line with Stokey (2012). Calibrating this parameter is motivated by the lack of human capital related data. Hence, a direct identification of δ^H is impossible. Although the structural model and the data indirectly provide information on human capital related parameters, the calibration reduces the degrees of freedom in favour of more conclusive inference on the common parameters. Additionally, I follow Kung and Schmid (2015) and set the depreciation rate of R&D to $\delta^N = 0.0375$.

3.2. Data

I obtain quarterly data from 1950 to 2016 for consumption, capital investment and GDP from the Bureau of Economic Analysis (BEA). Quarterly real return and T-Bill data are from the Center for Research in Security Prices (CRSP), and the consumer price index (CPI) is obtained from the Bureau of Labor Statistics (BLS). Consumption is constructed as the sum

[4]For details, refer to Goessling (2018).

[5]Note that the approach is related to the methods recently applied by Gust et al. (2017) who estimate a non-linear macroeconomic model by a MCMC approach. In contrast, the method applied in this paper is far more general and robust.

[6]Note that $\boldsymbol{\theta}$ denotes a set of parameters and should not be confused with the composite parameter θ of the utility function.

Parameter	Description	Prior/Calibration
Panel A: Estimated Parameters		
α	Physical capital share	$\mathcal{U}(0.26, 0.4)$
κ	Capital adjustment costs elasticity	$\mathcal{U}(0.1, 1)$
γ	Relative risk aversion	$\mathcal{U}(0, 15)$
ψ	Elasticity of intertemporal substitution	$\mathcal{U}(0.5, 4)$
δ^K	Physical capital depreciation rate	$\mathcal{U}(0, 0.1)$
η^N	Elasticity R&D capital	$\mathcal{U}(0.4, 0.8)$
η^H	Elasticity human capital	$\mathcal{U}(0.2, 0.8)$
ξ	Intermediate goods share	$\mathcal{U}(0.2, 0.8)$
ω	R&D weight	$\mathcal{U}(0.2, 0.8)$
χ	Elasticity of capital substitution	$\mathcal{U}(0, 1)$
σ	Productivity shocks volatility	$\mathcal{U}(0, 0.4)$
ρ	Productivity shock persistence	$\mathcal{U}(0.9, 1)$
lev	Leverage factor aggregate return	$\mathcal{U}(1.8, 4)$
$\sigma_{\Delta y}$	Measurement error volatility Δ_y	$\mathcal{N}^*(0.01, 0.1^2)$
$\sigma_{\Delta c}$	Measurement error volatility Δ_c	$\mathcal{N}^*(0.01, 0.1^2)$
$\sigma_{\Delta i}$	Measurement error volatility Δ_i	$\mathcal{N}^*(0.01, 0.1^2)$
σ_{rf}	Measurement error volatility Δ_{rf}	$\mathcal{N}^*(0.01, 0.1^2)$
$\sigma_{r^{eq}}$	Measurement error volatility $\Delta_{r^{eq}}$	$\mathcal{N}^*(0.01, 0.1^2)$
Panel B: Calibrated Parameters		
β	Time discount factor	0.996
g	Steady state growth rate	1.0045
δ^H	Human capital depreciation rate	0.04
δ^N	R&D capital depreciation rate	0.0375

Table 1: Prior Distributions and Calibrated Parameters. This table reports the prior distributions used for inference in Panel A. Furthermore, calibrated parameters are reported in Panel B. The remaining parameters are numerically calculated such that: the steady state growth rate equals 1.0045; the steady state of labor equals a third of available time; educational time equals a tenth of available time; and the balanced growth condition (9) holds. $\mathcal{U}(\cdot, \cdot)$ denotes a uniform distribution and $\mathcal{N}(\cdot, \cdot)$ is the normal distribution, where an asterisk denotes a truncation to the interval [0,0.1].

of non-durable consumption and services, investment from domestic fixed capital investment and output from GDP. The risk-free rate is constructed using the 90 days Treasury bond returns, and the equity return is proxied by the value-weighted total return market index. All nominal data series are deflating by the CPI.

The observation equations are linear and incorporate measurement errors,

$$\mathbf{y}_t = \begin{pmatrix} \Delta c_t \\ \Delta y_t \\ \Delta i_t \\ r_t^f \\ lev \cdot r_t^a \end{pmatrix} + \mathcal{N}\left(\mathbf{0}, diag\begin{pmatrix} \sigma_{\Delta c}^2 \\ \sigma_{\Delta y}^2 \\ \sigma_{\Delta i}^2 \\ \sigma_{r^f}^2 \\ \sigma_{r^{eq}}^2 \end{pmatrix}\right), \tag{21}$$

where Δ denotes growth rates of the model variables and $\mathbf{y}_t$ denotes the vector of observed variables. Furthermore, the $diag(x)$ operator denotes a quadratic matrix with diagonal elements x. Note that I explicitly account for leverage in the observed equity returns by introducing a leverage factor, such that the complete estimated parameter vector is $\boldsymbol{\theta} = (\alpha, \kappa, \gamma, \psi, \delta^K, \eta^N, \eta^H, \xi, \omega, \chi, \sigma, \rho, lev, \sigma_{\Delta i}, \sigma_{\Delta y}, \sigma_{\Delta c}, \sigma_{r^f}, \sigma_{r^{eq}})'$.

4. Results

4.1. Parameter Estimates and Model Moments

Table 2 reports the estimation results: the mean, mode and median as well as 90% Bayesian intervals. In light of the relatively small Bayesian intervals some model parameters are well identified, e.g. α, κ, γ, ψ, δ^K, and ρ. On the other hand, the intervals for the elasticities η^N and η^H, ξ and χ are relatively wide, and their posterior distributions are still close to their prior distributions. This is partly due to the lack of observable data about human capital and R&D capital, and partly due to the model inherent structure that allows to generate similar growth rates for consumption, output and so forth with different parameter combinations.

Note that the point estimates for some parameters slightly differ from the parameters used by Kung and Schmid (2015). This is intuitively clear, as the estimation approach by equation (21) adds an additional layer of measurement errors to the model variables. While a conventional calibration only matches chosen model moments, I jointly estimate the measurement error standard deviations along with the deep model parameters. As shown in Table 3 Column I, this results in a highly superior model fit, even though no moments are targeted initially. To calculate the model implies moments, I identify the $\boldsymbol{\theta}$ particle with highest posterior density and use it to simulate the model. Note that due to the mostly flat priors, this estimator virtually equals the maximum likelihood estimator. Table 3 contrasts the population moments obtained from 100 simulation runs of the model over 304 quarters with time aggregated statistics of the empirical data. Considering Column I in Table 3, the estimated model is able to reproduce the empirical moments. Notice that the simultaneously estimated parameters of the observation equation, Table 2 Panel B, allow to disentangle the endogenous model fit from the contribution of the measurement errors. In particular, Column II in Table 3 reports the moments excluding the exogenous error innovations. In line with the parameter estimates, the measurement error is negligible for consumption and output growth rates. In contrast, for investment growth, risk-free rate and equity return, measurement errors contribute roughly 35, 90 and 85 percent of the

Parameter	Mean	Mode	Median	Interval
Panel A: Model Parameters				
α	0.4488	0.4539	0.4501	[0.4004,0.4942]
κ	0.9416	0.9859	0.9559	[0.8393,0.9959]
γ	7.7903	8.7495	7.9253	[4.4960,10.6472]
ψ	2.3603	2.1085	2.2669	[1.7763,3.2893]
δ^K	0.0269	0.0144	0.0208	[0.0068,0.0689]
η^N	0.5776	0.5971	0.5752	[0.4238,0.7485]
η^H	0.5011	0.5765	0.5007	[0.2314,0.7659]
ξ	0.4052	0.2714	0.3701	[0.2157,0.7169]
w	0.7658	0.8733	0.7728	[0.5352,0.9781]
χ	0.2965	0.2254	0.2856	[0.0309,0.5958]
σ	0.1852	0.1227	0.1780	[0.0178,0.3721]
ρ	0.9901	0.9918	0.9908	[0.9811,0.9971]
Panel B: Measurement Parameters				
lev	1.9833	1.8324	1.9379	[1.8099,2.3084]
σ_{Δ_i}	0.0172	0.0172	0.0172	[0.0158,0.0188]
σ_{Δ_y}	0.0029	0.0034	0.0031	[0.0009,0.0044]
σ_{Δ_c}	0.0044	0.0043	0.0043	[0.0039,0.0048]
σ_{rf}	0.0089	0.0088	0.0089	[0.0083,0.0097]
$\sigma_{r^{eq}}$	0.0825	0.0819	0.0824	[0.0763,0.0891]

Table 2: Parameter Estimates. This table reports the mean, mode, median and a Bayesian 90% interval of the marginal posterior distributions. The mode is calculated from a kernel smoothing density estimator based on the respective parameter sample.

variation, respectively. This reflects two well known challenges for production based asset pricing models, namely too low an investment volatility, and negligible volatility in the risk free rate. Furthermore, the "pure" model generates too high positive correlations between the growth rates of the macroeconomic variables.

4.2. Bayesian Impulse Response Functions

Given the draws from the posterior distribution of $\boldsymbol{\theta}$, it is straightforward to construct Bayesian Impulse Response Functions (BIRF). For each draw $\boldsymbol{\theta}^i$, I compute the IRF to a one-standard-deviation shock of the exogenous process (1). Results are reported in Figure 1. The dark (light) grey areas correspond to pointwise the middle 50 (90) percent of the impulse responses.

The medians of the BIRF provide support for procyclical consumption (Panel A), procyclical expected consumption growth (Panel F), increasing education time (Panel E), and increasing labor hours (Panel B), i.e. households optimally decrease their leisure time to

	Data	Model	
		I	II
Panel A: Macro Moments			
$\sigma_{\Delta c}$ %	1.18	1.37	1.04
$\sigma_{\Delta i}$ %	4.88	4.55	2.99
$\sigma_{\Delta y}$ %	2.04	2.08	2.08
$\sigma_{\Delta c}/\sigma_{\Delta y}$	0.58	0.66	0.5
$\sigma_{\Delta i}/\sigma_{\Delta c}$	4.12	3.33	2.87
$\sigma_{\Delta i}/\sigma_{\Delta y}$	2.39	2.19	1.45
$corr[\Delta i, \Delta y]$	0.71	0.65	1
$corr[\Delta c, \Delta y]$	0.63	0.66	0.86
$corr[\Delta c, \Delta i]$	0.54	0.41	0.81
Panel B: Return Moments			
$\mathbb{E}[r^f]$ %	1.11	0.97	0.98
σ_{r^f} %	1.77	1.74	0.078
$\mathbb{E}[r^a]$ %	6.9	4.79	2.6
σ_{r^a} %	16.54	16.97	2.7
$\mathbb{E}[r^d]$ %	-	-	2.84
σ_{r^d} %	-	-	3.19

Table 3: Population Moments. This table reports annualized moments for the parameter vector $\boldsymbol{\theta}$ with the highest posterior density. The moments are calculated from 100 simulations of the model over 304 quarters. The first 80 quarters are dropped from the calculation of the population moments. Panel A shows the macroeconomic moments, and Panel B the return moments. The data column reports the empirical annualized moments. Column I reports the moments using the observation equation, while Column II reports the raw model moments.

work more and thus exploit a good economic environment. Additionally, households allocate more time to education in order to increase their future human capital. Thus, in contrast to the results of Wei (2005) or Dejong and Ingram (2001), the estimated model implies a procyclical education choice. This is in line with Malley and Woitek (2011) who investigate the empirical link between college enrollment and output, and find evidence in favor of procyclicality. Moreover, the education dynamics can be interpreted in the spirit of King and Sweetman (2002) who provide empirical evidence for *procyclical skill retooling* using Canadian administrative data.

4.3. Returns on Human Capital

Similarly to the BIRFs, I quantify the return on human capital by calculating population moments of the model for each draw of $\boldsymbol{\theta}$. Pooling the population moments allows to construct intervals for the mean and standard deviation of the return series conditional on the data. Table 4 shows the characteristics of human capital returns implied by the data set and the model.

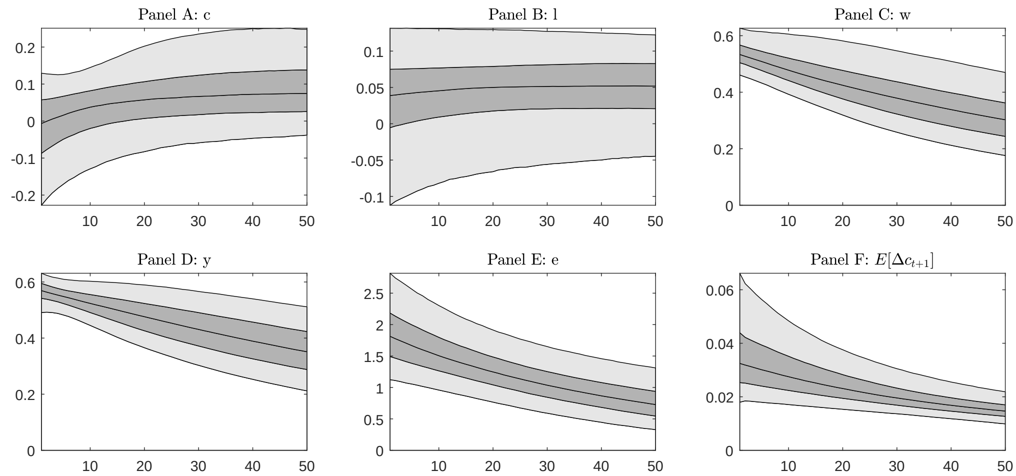

Figure 1: Bayesian Impulse Response Functions. This figure reports the BIRF of consumption (Panel A), labor (Panel B), wages (Panel C), output (Panel D), education (Panel E) and expected consumption growth (Panel F). Intervals are calculated pointwise and variables are in logs.

Moment	Mean	Interval
$\mathbb{E}[r^H]$	3.69	[2.6,5.0]
σ_{r^H}	5.38	[4.3,7.0]
$corr[r^H, r^a]$	0.043	[0.026,0.06]

Table 4: Human Capital Return Moments. This table reports the mean and a 90% interval of the annualized human capital return moments conditional on the posterior distribution of $\boldsymbol{\theta}$. The moments are calculated by simulating the model over 1000 quarters for each draw $\boldsymbol{\theta}^i$, where the first 250 quarters are dropped from the calculation of the moments.

In particular, the expected return of the unlevered human capital claim exhibits a mean of 3.69 percent with intervals of 2.6 to 5 percent across the posterior draws. The mean standard deviation is 5.38 with interval $[4.3, 7.0]$, and the correlation between the (levered) equity return and the human capital claim is small, but positive, with an absolute mean value of 0.043. The low correlation in the characteristics of the equity and the human capital return is partly due to the exogenous shocks in equation (21). Nevertheless, this result resembles the notion of human capital claims as a new and atypical asset class. Endogenizing the difference in characteristics constitutes an essential feature for future models.

5. Conclusion

In contrast to the extant literature, the human capital model presented in this paper matches key asset pricing facts. Further, it allows to study human capital accumulation and its return within a modern production based framework. Based on the data set, the estimated model quantifies the return on human capital in a range of about 2 to 5 percent. An interesting possible future extension is the inclusion of wage rigidities. As Donadelli and Grüning (2016) show, such an extension can improve the fit of the Kung and Schmid (2015) model to macroeconomic dynamics.

Apart from its model theoretic contributions, the paper adds to the literature by implementing an advanced, fully non-linear Bayesian estimation method to a modern production based asset pricing model. This approach is superior to a simple calibration of asset pricing models. A promising perspective for future work is to combine global solutions techniques (i.e. projection methods) with a detailed Bayesian analysis. On the empirical side, an obvious route for future research is to add observations equations related to the model parameters that are still calibrated.

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