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The effect of observables, functional specifications, model features and shocks on identification in linearized DSGE models

Sergey Ivashchenko [#] and Willi Mutschler [†]

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[†] Department of Economics, University of Münster, Germany

[#] The Institute of Regional Economy Problems, Russian Academy of Sciences, Russia;
Financial Research Institute, Ministry of Finance, Russia;
The faculty of Economics, Saint-Petersburg Stage University, Russia;
National Research University Higher School of Economics, St. Petersburg, Russia

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The effect of observables, functional specifications, model features and shocks on identification in linearized DSGE models

Sergey Ivashchenko[a,b,c,d], Willi Mutschler[e,*]

[a] *The Institute of Regional Economy Problems, Russian Academy of Sciences, Russia.*
[b] *Financial Research Institute, Ministry of Finance, Russia.*
[c] *The faculty of Economics, Saint-Petersburg State University, Russia.*
[d] *National Research University Higher School of Economics, St. Petersburg, Russia.*
[e] *Center for Quantitative Economics, University of Münster, Am Stadtgraben 9, 48143 Münster, Germany.*

Abstract

Both the investment adjustment costs parameters in Kim (2003) and the monetary policy rule parameters in An & Schorfheide (2007) are locally not identifiable. We show means to dissolve this theoretical lack of identification by looking at (1) the set of observed variables, (2) functional specifications (level vs. growth costs, output-gap definition), (3) model features (capital utilization, partial inflation indexation), and (4) additional shocks (investment-specific technology, preference). Moreover, we discuss the effect of these changes on the strength of parameter identification from a Bayesian point of view. Our results indicate that researchers should treat parameter identification as a model property, i.e. from a model building perspective.

Keywords: identification, weak identification, investment adjustment costs, Taylor rule, model features, shocks

JEL: C18, C51, C68, E22, E52

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*Corresponding author.

Email addresses: sergey.ivashchenko.ru@gmail.com (Sergey Ivashchenko), willi@mutschler.eu (Willi Mutschler)

1. Introduction

The identification problem in DSGE models. DSGE models have become a major toolkit for empirical macroeconomic research and an important policy tool used in central banks. Many different methods of solving and estimating DSGE models have been developed and used in order to obtain a detailed analysis and thorough estimation of dynamic macroeconomic relationships, see e.g. Fernández-Villaverde et al. (2016). Recently, the question of identifiability of DSGE models has proven to be of major importance, especially since the identification of a model precedes (consistent) estimation and inference of an unknown parameter vector $\theta \in \Theta$ from observations Y. θ_0 is said to be globally identified if for the family of probability distributions $p(Y|\theta)$ generated by a DSGE model: $p(Y|\theta) = p(Y|\theta_0)$ implies $\theta = \theta_0$. If this condition is only satisfied for values of θ in an open neighborhood of θ_0, then θ_0 is said to be locally identified. From an econometric point of view, local identification belongs to the usual regularity conditions and is necessary for the asymptotic theory of e.g. maximum likelihood estimation. From an economic point of view, lack of identification leads to wrong conclusions from calibration, estimation and inference (Canova & Sala, 2009), whereas the source of identification influences empirical findings (Ríos-Rull et al., 2012). Accordingly, experience shows that it is quite difficult both for Frequentist as well as Bayesian researchers to maximize the likelihood/posterior or minimize some (moment) objective function, because these functions are typically not well behaved as one has to deal with multiple local extrema, weak curvature in some directions of the parameter space and ridges. The evaluation of first-order and second-order derivatives is intractable and gradient based optimization methods perform quite poorly (Andreasen, 2010). The resulting estimators may often lie on the boundary of the theoretically admissible parameter space and conventional Gaussian asymptotics yield poor approximations to the true sampling distribution. In many cases the source of these particular outcomes is due to local or weak identifiability issues or an unfortunate choice of observables. Nevertheless, in particular Bayesian (and to some extent also Frequentist) estimation of DSGE models has rapidly progressed; however, the study of identifiability, which should precede estimation, is still a rather neglected topic for applied macroeconomists.

Research question. In this paper we seek to answer the question whether researchers should treat parameter identification as a model property, i.e. from a model building perspective. To this end, we revisit the linearized models of Kim (2003) and An & Schorfheide (2007), as it is well-known that both the multisectoral and intertemporal investment adjustment costs parameters in Kim (2003) and the monetary policy rule parameters in An & Schorfheide (2007) are locally not identifiable (Mutschler, 2016; Ratto & Iskrev, 2011). We try to dissolve this theoretical lack of identifiability by looking at (1) the set of observed variables, (2) functional specifications, (3) model features, and (4) additional shocks. Moreover, we discuss the effect of these changes on the strength of parameter identification from a Bayesian point of view.

Research methods. Regarding (1), we complement Canova et al. (2014) by selecting observables in a way that optimizes parameter identification. To this end, we are agnostic and analyze local identification for all possible combinations of observable variables. For (2), we focus on the functional specification of the intertemporal investment adjustment costs in the Kim (2003) model and of the output gap in the monetary policy rule of the An & Schorfheide (2007) model. With respect to (3), we analyze the effect on identification of capital utilization in the first model and of a partial inflation indexation scheme in the second model. Lastly regarding (4), we add an investment-specific technological shock into the former and a preference shock on the discount factor into the latter model. Our focus lies both on the theoretical properties as well as the strength of identification of the model parameters in linearized DSGE models. To this end, we solve the model using first-order perturbation techniques and carefully check the rank criteria of Iskrev (2010), Komunjer & Ng (2011) and Qu & Tkachenko (2012) for all considered model variants and observables. Regarding the strength of identification we make use of Koop et al. (2013)'s Bayesian learning rate indicator.

Outline. In section 2, we summarize our implementation of the used tools to check for parameter identification in linearized DSGE models. We analyze the identification properties of the Kim (2003) model in section 3, whereas section 4 provides the corresponding analysis for the An & Schorfheide (2007) model. In section 5, we discuss our results from a model building perspective. Section 6 concludes. The replication files are available in a GitHub repository (`https://github.com/wmutschl/identification-note/`).

2. Implementation of identification checks

Rank checks. We check the local identification properties according to Iskrev (2010), Komunjer & Ng (2011) and Qu & Tkachenko (2012) with Dynare (Adjemian et al., 2011). In a nutshell, Iskrev (2010)'s approach to find non-identified parameters is based on observational equivalent moments, i.e. on the sensitivity of the theoretical mean and autocovariances, whereas Qu & Tkachenko (2012)'s approach focuses on observational equivalent spectral properties, i.e. on the sensitivity of the theoretical mean and spectrum of observables. Komunjer & Ng (2011) study the implications of observational equivalence in minimal systems and derive a finite system of nonlinear equations that admits a unique solution if and only if the parameters are identified. In all three cases, we need to compute Jacobians w.r.t the model parameters and check whether these have full rank. Iskrev (2010) follows an analytical approach to compute the Jacobian of moments using Kronecker products, which is extended in Ratto & Iskrev (2011) by making use of computationally more efficient generalized Sylvester equations. Both Komunjer & Ng (2011) and Qu & Tkachenko (2012), however, rely on numerical methods to compute the derivatives of the minimal system and spectrum, which is known to be sensitive to the thresholds and tolerance levels used. We are, however, able to extend the ideas from Iskrev (2010) and Ratto & Iskrev (2011) to also compute these Jacobians analytically. Hence, we extend the identification toolbox of Dynare such that users are able to compute all three rank criteria (moments, minimal system or spectrum) analytically (by either using Kronecker products or generalized Sylvester equations).[1] To pinpoint the problematic parameters that yield rank failure, the default in Dynare is to look into the nullspace and evaluate multicorrelation coefficients of the columns. Another (numerically more robust) approach, which is contributed to the software and used in this paper, is to check the ranks for all possible combinations of parameters, and to mark the ones that do not pass the rank check.

[1] Our contributions and improvements are already merged into the 4.6-unstable branch. For more computational details, see the merge request (Mutschler, 2019).

Bayesian learning rate indicator. Koop et al. (2013)'s Bayesian learning rate indicator is based on a Bayesian simulation approach that looks at the posterior average precision of the parameters, i.e. the inverse of the posterior variance divided by the sample size T. The posterior precision should increase at a rate of T for identified parameters, whereas for weakly identified parameters it increases at a slower rate. In other words, the average precision of a strongly identified parameter should tend to a constant, whereas for a weakly identified parameter it is heading quickly towards zero. We generate one artificial dataset of 50000 observations and then estimate the parameters with Bayesian MCMC methods using the first T = 100, 300, 900, 2700, and 8100 of the simulated observations. Then, on the one hand, we follow the approach in Chadha & Shibayama (2018) and compute the average posterior precision by taking the inverse of the product of the posterior variance times T and examine if it converges to a constant, suggesting the posterior precision is updated at the same rate as T. On the other hand, we also compute convergence ratios as in Kamber et al. (2016); that is, we compare the ratio of two subsequent estimated posterior precision values, e.g. at T=100 and T=300, and check whether this ratio is close to the rate at which T increases, i.e. close to 300/100=3.

3. Investment adjustment costs model

The Kim (2003) model is a variant of the canonical Real Business Cycle model with log utility, however, extended by two kinds of investment adjustment costs. First, *multisectoral adjustment costs*, governed by a parameter θ, enter the budget constraint:

$$\underbrace{\left[(1-SAV)\left(\frac{C_t}{1-SAV}\right)^{1+\theta}+SAV\left(\frac{I_t}{SAV}\right)^{1+\theta}\right]^{\frac{1}{1+\theta}}}_{:=Y_t^d}=R_t^K U_t^K K_{t-1}-\Psi_t^K K_{t-1} \quad (1)$$

where C_t is consumption, I_t is investment, and SAV denotes the steady state savings rate, $SAV = \frac{I}{Y^d}$. Similar to Huffman & Wynne (1999) we focus on $\theta > 0$, i.e. a reverse CES technology, in order for the production possibilities set to be convex. Note that for $\theta = 0$ the transformation reduces to the standard linear case, $Y_t^d = C_t + I_t$. Different to Kim (2003), we introduce a cost, Ψ_t^K, of capital utilization per unit of physical capital.[2] U_t^K denotes the capital utilization rate and physical (end-of-period) capital, K_t, is transformed into effective (end-of-period) capital, K_t^s, according to $K_{t-1}^s = U_t^K K_{t-1}$. Effective capital is then rented to the representative firm at the gross rental rate R_t^K. The firm produces a homogeneous good using a Cobb-Douglas production function, $Y_t = A_t(K_{t-1}^s)^\alpha$, where A_t denotes total factor productivity.

Second, *intertemporal adjustment costs*, governed by a parameter κ, are introduced into the capital accumulation equation, which involve a nonlinear substitution between the capital stock and investment. We consider two different specifications for this friction:

$$K_t = \left[(1-\delta)K_{t-1}^{1-\kappa} + \delta\left(\frac{\upsilon_t I_t}{\delta}\right)^{1-\kappa}\right]^{\frac{1}{1-\kappa}} \tag{2a}$$

$$K_t = (1-\delta)K_{t-1} + \upsilon_t I_t\left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) \tag{2b}$$

where δ denotes the depreciation rate. The first one, which we call the LEVEL specification, is also used by Kim (2003). It is based on Lucas & Prescott (1971) and involves costs in terms of the first derivative of capital or, in other words, on the current level of investment. The second one, which we call the GROWTH specification, is based on Christiano et al. (2005) and involves costs in terms of investment changes between periods. [3] Note that for $\kappa = 0$ we get the usual linear capital accumulation specification, i.e. $K_t = (1-\delta)K_{t-1} + \upsilon_t I_t$, in both cases. Different to Kim (2003), we introduce an investment-specific technological change, υ_t, in the fashion of Greenwood et al. (2000) and Justiniano et al. (2010). Both the log of A_t and the log of υ_t evolve according to AR(1) processes with persistence ρ_j and additive shocks, $\varepsilon_{j,t}$, which are assumed to be normally distributed with zero mean and standard deviation σ_j ($j = A, \upsilon$).

[2] To this end, we use the following functional form: $\Psi_t^K = (1-\psi_K)(U_t^K - U^K) + \frac{\psi_K}{2}(U_t^K - U^K)^2$, $\Psi_t^{K'} = (1-\psi_K) + \psi_K(U_t^K - U^K)$, $\Psi_t^{K''} = \psi_K$, such that the usual steady state normalization, $\Psi^{K''}/\Psi^{K'} = \psi_K/(1-\psi_K)$, applies.

[3] Regarding the functional form, we follow Schmitt-Grohé & Uribe (2004b) and set $S_t := S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$, $S_t' = \kappa\left(\frac{I_t}{I_{t-1}} - 1\right)$, $S_t'' = \kappa$ such that the usual steady state normalization, $S(1) = 0$, $S'(1) = 0$ and $S''(1) > 0$, applies.

The model equations can be summarized by:

$$\left(\frac{C_t}{(1-SAV)\cdot Y_t^d}\right)^{\theta}\Lambda_t = C_t^{-1} \tag{3}$$

$$\left(\frac{I_t}{SAV\cdot Y_t^d}\right)^{\theta}\Lambda_t = \begin{cases}\Lambda_t Q_t\left(\frac{\delta K_t}{\upsilon_t I_t}\right)^{\kappa}\\ \Lambda_t Q_t\left(1-S_t-\left(\frac{I_t}{I_{t-1}}\right)S_t'\right)\upsilon_t+\beta E_t\left[\Lambda_{t+1}Q_{t+1}\upsilon_{t+1}\left(\frac{I_{t+1}}{I_t}\right)^2 S_{t+1}'\right]\end{cases} \tag{4}$$

$$\Lambda_t Q_t = \begin{cases}\beta E_t\left[\Lambda_{t+1}\left(R_{t+1}^K U_{t+1}^K-\Psi_{t+1}^K+(1-\delta)Q_{t+1}\left(\frac{K_{t+1}}{K_t}\right)^{\kappa}\right)\right]\\ \beta E_t\left[\Lambda_{t+1}\left(R_{t+1}^K U_{t+1}^K-\Psi_{t+1}^K+(1-\delta)Q_{t+1}\right)\right]\end{cases} \tag{5}$$

$$R_t^K = \Psi_t^{K\prime} \tag{6}$$

$$R_t^K = \frac{\alpha Y_t}{K_t^s} \tag{7}$$

$$Y_t = Y_t^d + \Psi_t^K K_{t-1} \tag{8}$$

$$\log A_t = \rho_A \log A_{t-1} + \sigma_A \varepsilon_t^A \tag{9}$$

$$\log \upsilon_t = \rho_\upsilon \log \upsilon_{t-1} + \sigma_\upsilon \varepsilon_t^\upsilon \tag{10}$$

where Λ_t denotes the Lagrange multiplier corresponding to (1) and $\Lambda_t Q_t$ to (2). The upper part of equations (4) and (5) correspond to the LEVEL specification of intertemporal investment adjustment costs, whereas the lower part is associated with the GROWTH specification. The steady state is given by normalizations, $A = Q = U^K = \upsilon = 1$, and equations $R^K = \left(\frac{1}{\beta}+\delta-1\right)\frac{Q}{U^K}$, $K = \left(\frac{\alpha A}{R^K}\right)^{\frac{1}{1-\alpha}}$, $I = \frac{\delta K}{\upsilon}$, $Y = AK^{\alpha}$, $C = (1-SAV)Y$ and $\Lambda = C^{-1}$. Our calibration and prior specification of parameters is given in table 1. The calibration of α, β, and δ is based on a steady state investment to output ratio, I/Y, of 0.25, a steady state capital productivity, K/Y, of 10 and an annualized steady state interest rate, R^A, of 2. ψ_K is implicitly defined via the first order necessary conditions with respect to K_t and U_t^K. θ and κ are based on values taken from Ratto & Iskrev (2011), whereas the parameters of the stochastic processes are chosen symmetrically with mild persistence and amplitude of shocks.

To check the sensitivity of local identifiability to changes in observables, model assumptions, functional specifications, and shocks, we distinguish three different model scenarios and consider all possible one- and two-set combinations of model variables as observables. Our focus lies on observable variables that are commonly used in the lit-

Table 1: Parameters, priors and bounds for investment adjustment costs model

Parameters		*Bounds*		*Prior Specification*		
Symbol	*Value*	*Lower*	*Upper*	*Density*	*Para (1)*	*Para (2)*
θ	1.5	1e-8	10	Gamma	1.5	0.75
κ	2	1e-8	10	Gamma	2	1.5
α	0.3	1e-8	0.9999	Normal	0.3	0.05
δ	0.025	1e-8	0.9999	Uniform	0	1
r^A	2	1e-8	10	Gamma	2	0.25
ρ_A	0.5	1e-8	0.9999	Beta	0.5	0.1
σ_A	0.6	1e-8	10	Inverse Gamma	0.6	4
ψ_K	0.97	1e-8	0.9999	Uniform	0	1
ρ_υ	0.5	1e-8	0.9999	Beta	0.5	0.1
σ_υ	0.6	1e-8	10	Inverse Gamma	0.6	4

Notes: Para (1) and (2) list the means and the standard deviations for Beta, Gamma, and Normal distributions. For the Inverse Gamma distribution Para (1) and (2) equal s and v, where $\wp_{IG}(\sigma|v,s) \propto \sigma^{-v-1}e^{-vs^2/2\sigma^2}$, whereas for the Uniform distribution these correspond to the lower and upper bound. The effective prior is truncated at the boundary of the determinacy region.

erature; namely, output, consumption, investment, and the return of capital.[4] Our first scenario, called BASELINE, corresponds to the original model specification of Kim (2003). Accordingly, we switch off both capital utilization and investment-specific technological change. In our second scenario, called CAPITAL UTILIZATION, we analyze the effect on local identification of adding capital utilization costs to the BASELINE scenario. Likewise, in our third scenario, called INVESTMENT SHOCK, we add investment-specific technological change to the BASELINE case. Note that in the first two scenarios there is only one shock in the model, whereas in the last scenario there are two. Lastly, each scenario is run with either the LEVEL or GROWTH specification of intertemporal investment adjustment costs.

Table 2 summarizes whether the required rank conditions are fulfilled for the different scenarios and combinations of observable variables. As expected and analytically shown by Kim (2003), θ and κ cannot be identified jointly in the BASELINE scenario with the LEVEL specification. The GROWTH specification, however, allows one to identify these parameters for many choices of feasible observable variables. In particular, among

[4]For the sake of completeness, we also analyze (usually unobserved) variables like technology, capital or the auxiliary Lagrange multipliers. We like to point out that by observing marginal utility Λ_t or Tobin's Q_t combined with any other variable, one is able to locally identify all model parameters independent of the specification of intertemporal investment adjustment costs or model scenario. Observing capital or technology, on the other hand, does not solve the lack of identification in the considered scenarios. The exact results can be found in the replication files.

single observable variables, consumption C_t (and not Y_t) yields a locally fully identified model. Intuitively, the GROWTH specification adds another state variable into the model in terms of lagged investment. The coefficients of lagged investment in the decision rules will depend on the intertemporal adjustment costs parameter in a manner that is distinct from the multisector adjustment costs. Hence, we can distinguish the dynamics of multisectoral *level* adjustment costs from intertemporal *growth* adjustment costs. Our other two scenarios, CAPITAL UTILIZATION and INVESTMENT SHOCK, show further means to identify κ and θ. Almost all pairs of variables yield full rank in these scenarios, even when considering the LEVEL specification. But, of course, this also holds for the GROWTH specification. Note that single observable variables fail to identify all model parameters as κ and θ are co-linear with either the capital utilization or the investment shock process parameters.

Table 2: Rank checks for investment adjustment costs model

	Baseline						Capital Utilization						Investment Shock					
	Level			Growth			Level			Growth			Level			Growth		
	MOM	MIN	SPEC	MOM	MIN	SPEC	MOM	MIN	SPEC	MOM	MIN	SPEC	MOM	MIN	SPEC	MOM	MIN	SPEC
Y	$[\kappa\theta]$	ERR	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	ERR	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	ERR	$[\kappa\theta]$	$[\kappa\theta]$	ERR	$[\kappa\theta]$
C	$[\kappa\theta]$	ERR	$[\kappa\theta]$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$[\kappa\theta]$	ERR	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	ERR	$[\kappa\theta]$	$[\kappa\theta]$	ERR	$[\kappa\theta]$
I	$[\kappa\theta]$	ERR	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	ERR	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	ERR	$[\kappa\theta]$	$[\kappa\theta]$	ERR	$[\kappa\theta]$
R^K	$[\kappa\theta]$	ERR	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	ERR	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa]$	$[\kappa\theta]$	ERR	$[\kappa\theta]$	$[\kappa\theta]$	ERR	$[\kappa\theta]$
K	$[\kappa\theta]$	ERR	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	ERR	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	ERR	$[\kappa\theta]$	$[\kappa\theta]$	ERR	$[\kappa\theta]$
Λ	$[\kappa\theta]$	ERR	$[\kappa\theta]$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$[\kappa\theta]$	ERR	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	ERR	$[\kappa\theta]$	$[\kappa\theta]$	ERR	$[\kappa\theta]$
Q	$[\kappa\theta]$	ERR	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	ERR	$[\kappa\theta]$	$[\kappa\theta]$	ERR	$[\kappa\theta]$	$[\kappa\theta]$	ERR	$[\kappa\theta]$	$[\kappa\theta]$	ERR	$[\kappa\theta]$
A	$[\kappa\theta]$	ERR	$[\kappa\theta]$	$[\kappa\theta]$	ERR	$[\kappa\theta]$	$[\kappa\theta]$	ERR	$[\kappa\theta]$	$[\kappa\theta]$	ERR	$[\kappa\theta]$	$[\kappa\theta]$	ERR	$[\kappa\theta]$	$[\kappa\theta]$	ERR	$[\kappa\theta]$
U^K	-	-	-	-	-	-	$[\kappa\theta]$	ERR	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	-	-	-	-	-	-
υ	-	-	-	-	-	-	-	-	-	-	-	-	$[\kappa\theta]$	err	$[\kappa\theta]$	$[\kappa\theta]$	err	$[\kappa\theta]$
Y, C	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$
Y, I	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$
Y, R^K	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$
C, I	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$
C, R^K	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$
I, R^K	$[\kappa\theta]$	$[\kappa\theta]$	$[\kappa\theta]$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$	$\checkmark\checkmark$

Notes: MOM corresponds to Iskrev (2010)'s, MIN to Komunjer & Ng (2011)'s, and SPEC to Qu & Tkachenko (2012)'s rank criteria. A $\checkmark\checkmark$ indicates that all model parameters are theoretically identifiable. $[\kappa\theta]$ indicates that both κ and θ cannot be identified jointly or are co-linear with respect to other parameters, whereas a single $[\kappa]$ or $[\theta]$ implies non-identification of that parameter. ERR indicates that the criteria cannot be computed (mostly the order condition is not met), whereas a - indicates that this set of variables is not available in the specific scenario.

Moreover, in the replication files we also consider the effect of a different utility function, internal or external habit, labor choice and monetary policy rules on parameter identification of θ and κ. We briefly summarize our findings. A CRRA utility function or the inclusion of internal/external habit formation does not change the above results.[5] The inclusion of labor (as already shown by Kim (2003)) facilitates parameter identification of θ and κ in both cases, but adds other parameters that can only be identified by observing either hours or wages. Extending the BASELINE model with respect to bond holdings requires the inclusion of a Taylor rule. This also provides means for identifying the investment adjustment costs parameters in both the LEVEL and GROWTH specification, however, for several combinations of observables the parameters of the monetary rule are not identified, a topic we study in more detail in the next section.

Tables 3 and 4 provide insight into the strength of identification according to the Bayesian learning rate indicator of Koop et al. (2013) for the BASELINE scenario with observable C_t and the INVESTMENT SHOCK scenario with observable Y_t and C_t.[6] The simulation and estimation exercise reveals that the strength of identification of the investment adjustment costs parameters, θ and κ, is weak in both BASELINE scenarios as well as the INVESTMENT SHOCK LEVEL case, since the rates at which the posterior precisions are updated are slower than the sample size change. In other words, the average posterior precision values in the table tend towards zero instead of a constant value. This is also evident by looking at the convergence ratios in table 4 as these stay close to 1 and do not tend towards the change in samples size of 3. The INVESTMENT SHOCK GROWTH specification, however, is the exception, as κ and θ are both strongly identifiable: The average precisions tend towards a constant and the convergence ratios fluctuate around 3. Regarding the other model parameters we find mixed results. In all cases under consideration the strength of identification of R^A (and hence β) is weak, which is a common finding in the literature (Morris, 2017). In the (unidentified) BASELINE LEVEL scenario we see that only ρ_A is strongly identifiable. This confirms that estimating non-identified models yields severe problems in the estimation of other, actually identified model pa-

[5] In some cases we find that the identification criteria of Iskrev (2010), Komunjer & Ng (2011) and Qu & Tkachenko (2012) yield different results. We experimented with the settings and found that the differences are driven by numerical thresholds, tolerance levels and the method used to normalize the Jacobians for rank computations.

[6] We choose these scenarios due to the fact that our focus is on applied researchers who use Dynare for Bayesian estimation. Accordingly, we do not analyze the strength of identification in the CAPITAL UTILIZATION scenario as this requires techniques to estimate singular DSGE models, which cannot be done with Dynare out-of-the-box (yet).

rameters. Accordingly, in the (identified) BASELINE GROWTH scenario α, δ, ρ_A and σ_A are (more or less) strongly identified. Likewise, the GROWTH specification in the INVESTMENT SHOCK scenarios is better than the LEVEL one as the convergence ratios are closer to 3.

Table 3: Average posterior precisions for investment adjustment costs model

	Parameters								
T	α	R^A	δ	ρ_A	σ_A	θ	κ	ρ_v	σ_v
	BASELINE LEVEL, OBSERVABLE $[C_t]$								
100	5.02419	0.16006	68.06352	2.35966	2.23979	0.01789	0.00460	-	-
300	1.66330	0.05318	10.19060	1.66202	0.37313	0.00576	0.00227	-	-
900	0.60317	0.01777	4.79942	1.37207	0.24600	0.00194	0.00066	-	-
2700	0.23070	0.00594	2.50897	1.28453	0.16133	0.00066	0.00015	-	-
8100	0.10292	0.00199	1.48975	1.29131	0.06690	0.00023	0.00005	-	-
	BASELINE GROWTH, OBSERVABLE $[C_t]$								
100	5.09980	0.15940	68.82651	2.13940	2.46914	0.01751	0.00422	-	-
300	1.96554	0.05286	12.74101	1.40185	0.40590	0.00587	0.00142	-	-
900	0.71330	0.01779	5.90759	0.85181	0.28399	0.00198	0.00043	-	-
2700	0.32161	0.00592	3.57656	0.57919	0.19469	0.00072	0.00018	-	-
8100	0.21194	0.00197	3.67273	0.35742	0.09412	0.00021	0.00006	-	-
	INVESTMENT SHOCK LEVEL, OBSERVABLE $[Y_t, C_t]$								
100	79.05959	0.16094	225.54380	2.22210	0.90266	0.01976	0.05555	2.04271	0.72635
300	60.78898	0.05334	200.61253	1.53605	1.04589	0.00603	0.03482	1.57010	0.22718
900	36.43431	0.01788	200.06671	1.36422	1.16714	0.00224	0.01378	1.39247	0.12846
2700	17.94476	0.00602	99.71930	1.34674	0.80680	0.00078	0.00408	1.32375	0.06071
8100	7.11469	0.00205	61.69216	1.33956	0.83797	0.00036	0.00190	1.32053	0.02746
	INVESTMENT SHOCK GROWTH, OBSERVABLE $[Y_t, C_t]$								
100	71.67930	0.16314	334.96703	2.33540	0.95497	0.18745	0.18640	1.85807	1.25801
300	58.08013	0.05277	207.52371	1.57582	0.87680	0.06891	0.11989	1.38878	0.48674
900	36.47654	0.01776	227.33669	1.38348	1.07441	0.04741	0.09728	1.30844	0.43364
2700	17.90416	0.00615	116.05847	1.35925	0.74264	0.05246	0.09988	1.27667	0.39086
8100	7.74727	0.00244	85.15640	1.35892	0.82375	0.04136	0.08509	1.26054	0.34923

Notes: Posterior precisions are computed from the draws of the marginal posterior distributions given by a Random-Walk Metropolis-Hastings (RWMH) sampling algorithm based on four Markov chains with each 1000000 draws, half are being discarded as burn-in draws in each chain. The mode and Hessian evaluated at the mode (computed by Dynare's `mode_compute = 4`, i.e. Chris Sims's CSMINWEL) are used to determine the initial Gaussian proposal density with scale parameter set such that the acceptance ratios lie in between 20%-35%.

To sum up, in all our experiments we found that the GROWTH specification is superior to the LEVEL specification in terms of theoretical identification. Moreover, investment-specific technological change improves the strength of model parameters. Therefore, we provide theoretical support (from an identification point-of-view) for using both Christiano et al. (2005)'s GROWTH specification of investment adjustment costs and Greenwood et al. (2000)'s investment-specific technological change in modern DSGE models.

Table 4: Convergence ratios for posterior precisions for investment adjustment costs model

	Parameters								
$T/T(-1)$	α	R^A	δ	ρ_A	σ_A	θ	κ	ρ_υ	σ_υ
	BASELINE LEVEL, OBSERVABLE $[C_t]$								
300/100	0.993	0.997	0.449	2.113	0.500	0.966	1.479	-	-
900/300	1.088	1.003	1.413	2.477	1.978	1.009	0.868	-	-
2700/900	1.147	1.002	1.568	2.809	1.967	1.017	0.693	-	-
8100/2700	1.338	1.004	1.781	3.016	1.244	1.063	0.927	-	-
	BASELINE GROWTH, OBSERVABLE $[C_t]$								
300/100	1.156	0.995	0.555	1.966	0.493	1.005	1.010	-	-
900/300	1.089	1.010	1.391	1.823	2.099	1.012	0.903	-	-
2700/900	1.353	0.998	1.816	2.040	2.057	1.095	1.230	-	-
8100/2700	1.977	0.998	3.081	1.851	1.450	0.892	1.102	-	-
	INVESTMENT SHOCK LEVEL, OBSERVABLE $[Y_t, C_t]$								
300/100	2.307	0.994	2.668	2.074	3.476	0.915	1.881	2.306	0.938
900/300	1.798	1.006	2.992	2.664	3.348	1.113	1.187	2.661	1.696
2700/900	1.478	1.011	1.495	2.962	2.074	1.041	0.889	2.852	1.418
8100/2700	1.189	1.020	1.856	2.984	3.116	1.403	1.399	2.993	1.357
	INVESTMENT SHOCK GROWTH, OBSERVABLE $[Y_t, C_t]$								
300/100	2.431	0.970	1.859	2.024	2.754	1.103	1.930	2.242	1.161
900/300	1.884	1.010	3.286	2.634	3.676	2.064	2.434	2.826	2.673
2700/900	1.473	1.039	1.532	2.947	2.074	3.320	3.080	2.927	2.704
8100/2700	1.298	1.188	2.201	2.999	3.328	2.365	2.556	2.962	2.680

Notes: Posterior precisions are computed from the draws of the marginal posterior distributions given by a Random-Walk Metropolis-Hastings (RWMH) sampling algorithm based on four Markov chains with each 1000000 draws, half are being discarded as burn-in draws in each chain. The mode and Hessian evaluated at the mode (computed by Dynare's `mode_compute = 4`, i.e. Chris Sims's CSMINWEL) are used to determine the initial Gaussian proposal density with scale parameter set such that the acceptance ratios lie in between 20%-35%.

4. Monetary model

The An & Schorfheide (2007) model is a prototypical New Keynesian DSGE model and consists of a representative household purchasing a basket of differentiated goods using a Dixit-Stiglitz type aggregator and supplying homogeneous labor services. The differentiated goods are supplied by monopolistically competitive firms using only labor services within a linear production function. Each firm sets prices according to the Rotemberg pricing assumption such that changing prices entails a real cost in terms of goods. Labor productivity, A_t, is the driving force of the economy and evolves according to a unit root process, i.e. $\log(A_t/A_{t-1}) = \log(\gamma) + \log(z_t)$, where γ denotes the steady state growth rate of the economy. Hence, $y_t = Y_t/A_t$ denotes detrended output and $c_t = C_t/A_t$ detrended consumption. The monetary authority follows a Taylor rule for the nominal interest rate R_t and real government spending G_t is assumed to evolve stochastically as a ratio of output $g_t := (1 - G_t/Y_t)^{-1}$. Uncertainty is introduced via random fluctuations in productivity growth, government spending and a monetary policy shock.

We extend the model in three (common) directions. First, we add a preference shock, ζ_t, to the utility function that shifts the discount factor in the intertemporal optimization problem of the household without changing the intratemporal labor supply decision. Therefore, the Lagrange multiplier corresponding to marginal utility is given by $\lambda_t = \zeta_t c_t^{-\tau}$. Second, the Rotemberg price adjustment function of the j-th intermediate firm, $ac_t(j) = \frac{\phi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - \Gamma_{t,t-1}\right)^2 y_t(j)$, follows either a full, $\Gamma_{t,t-1} = \pi^*$, or a partial, $\Gamma_{t,t-1} = \pi_t^{\iota^p} \pi^{*1-\iota^p}$, indexation scheme, where π^* denotes target inflation. The first scheme corresponds to the original specification of An & Schorfheide (2007), whereas the second one is in the fashion of Smets & Wouters (2007). Third, we consider four different monetary policy rules that differ in the definition of the output gap: (i) deviation from the output value under flexible prices but with the monopoly power distortion intact, (ii) deviation from the steady state value of output, (iii) deviation from the growth trend, and (iv) the Smets & Wouters (2007) rule which combines (i) with differences in growth rates of output and the flex-price output.

$$R_t^*/R = (\pi_t/\pi^*)^{\psi_\pi} (y_t/y_t^*)^{\psi_y} \qquad \text{(flex-price)} \tag{11a}$$

$$R_t^*/R = (\pi_t/\pi^*)^{\psi_\pi} (y_t/y)^{\psi_y} \qquad \text{(steady state)} \tag{11b}$$

$$R_t^*/R = (\pi_t/\pi^*)^{\psi_\pi} (z_t \cdot y_t/y_{t-1})^{\psi_y} \qquad \text{(growth)} \tag{11c}$$

$$R_t^*/R = (\pi_t/\pi^*)^{\psi_\pi} (y_t/y_t^*)^{\psi_y} \left(\frac{y_t/y_{t-1}}{y_t^*/y_{t-1}^*}\right)^{\psi_{\Delta y}} \qquad \text{(SW)} \tag{11d}$$

where $y_t^* = (1-\nu)^{\frac{1}{\tau}} g_t$ is the output under flexible prices ($\phi = 0$) but with the monopoly power distortion intact. All shocks, $\varepsilon_{j,t}(j = R, g, z, \zeta)$, are assumed to be normally distributed with zero mean and standard deviation σ_j.

The model equations can be summarized by:

$$YGR_t = \gamma^Q + 100\left[\log\left(\frac{y_t}{y_{t-1}}\right) + \log\left(\frac{z_t}{z}\right)\right] \tag{12}$$

$$INFL_t = \pi^A + 400\log\left(\frac{\pi_t}{\pi}\right) \tag{13}$$

$$INT_t = \pi^A + r^A + 4\gamma^Q + 400\log\left(\frac{r_t}{r}\right) \tag{14}$$

$$\lambda_t = \beta\lambda_{t+1}\frac{R_t}{\gamma z_{t+1}\pi_{t+1}} \tag{15}$$

$$1 = \frac{1}{\nu}(1-\lambda_t^{-1}) + \frac{\phi}{2}(\pi_t - \Gamma^p_{t,t-1}) - \frac{\phi}{2\nu}(\pi_{t+1} - \Gamma^p_{t+1,t})^2 + \phi\beta\frac{\lambda_{t+1}}{\lambda_t}\frac{y_{t+1}}{y_t}(\pi_{t+1} - \Gamma^p_{t+1,t})\pi_{t+1} \tag{16}$$

$$y_t = c_t + \left(1 - \frac{1}{g_t}\right)y_t + \frac{\Phi}{2}(\pi_t - \Gamma^p_{t,t-1})^2 y_t \tag{17}$$

$$\log(g_t) = (1-\rho_g)\log(g) + \rho_g\log(g_{t-1}) + \varepsilon_{g,t} \tag{18}$$

$$\log(z_t) = \rho_z\log(z_{t-1}) + \varepsilon_{z,t} \tag{19}$$

$$\log(\zeta_t) = \left(1-\rho_\zeta\right)\log(\zeta) + \rho_\zeta\log(\zeta_{t-1}) + \varepsilon_{\zeta,t} \tag{20}$$

$$R_t = {R_t^*}^{1-\rho_R} R_{t-1}^{\rho_R}\exp\left\{\varepsilon_{R,t}\right\} \tag{21}$$

Note that $\gamma = 1+\gamma^Q/100$, $\beta = (1+r^A/400)^{-1}$, and $\pi^* = 1+\pi^A/400$. The calibration,

Table 5: Parameters, priors and bounds for monetary model

Parameters		*Prior Specification*			*Bounds*	
Symbol	*Value*	*Density*	*Para (1)*	*Para (2)*	*Lower*	*Upper*
r^A	1.00	Gamma	0.80	0.50	1e-5	10
π^A	3.20	Gamma	4.00	2.00	1e-5	20
γ^Q	0.55	Normal	0.40	0.20	-5	5
τ	2.00	Gamma	2.00	0.50	1e-5	10
ν	0.10	Beta	0.10	0.05	1e-5	0.99999
ψ_π	1.50	Gamma	1.50	0.25	1e-5	10
ψ_y	0.125	Gamma	0.50	0.25	1e-5	10
$\psi_{\Delta y}$	0.2	Gamma	0.20	0.15	1e-5	10
ρ_R	0.75	Beta	0.50	0.20	1e-5	0.99999
ρ_g	0.95	Beta	0.80	0.10	1e-5	0.99999
ρ_z	0.90	Beta	0.66	0.15	1e-5	0.99999
$100\sigma_R$	0.2	InvGamma	0.30	4.00	1e-8	5
$100\sigma_g$	0.6	InvGamma	0.40	4.00	1e-8	5
$100\sigma_z$	0.3	InvGamma	0.40	4.00	1e-8	5
ϕ	50	-	-	-	-	-
c/y	0.85	-	-	-	-	-
ι^p	0.5	Beta	0.50	0.15	1e-8	1
ρ_ζ	0.75	Beta	0.50	0.20	1e-5	0.99999
$100\sigma_\zeta$	0.2	InvGamma	0.30	4.00	1e-8	5

Notes: Para (1) and (2) list the means and the standard deviations for Beta, Gamma, and Normal distributions. For the Inverse Gamma distribution Para (1) and (2) equal s and v, where $\wp_{IG}(\sigma|v,s) \propto \sigma^{-v-1}e^{-vs^2/2\sigma^2}$. The effective prior is truncated at the boundary of the determinacy region. Note that $g^* = (c/y)^{-1}$.

priors and bounds for the model parameters are given in table 5 and are taken from An & Schorfheide (2007) and Smets & Wouters (2007). Note that Mutschler (2015) shows that the set of parameters (ν, ϕ) and steady state ratio $g^* = \frac{y}{c}$ do not enter the linearized solution and are only identifiable via a higher-order approximation of the policy functions. Therefore, we fix g^* and ϕ throughout our analysis. The steady state is given by normalizations, $z = \zeta = 1$, $\pi = \pi^*$, $g = g^*$, and equations $r = \frac{\gamma z \pi}{\beta}$, $c = (1-\nu)^{\frac{1}{\tau}}$, $y = g \cdot c$, $YGR = \gamma^Q$, $INFL = \pi^A$, $INT = \pi^A + r^A + 4\gamma^Q$.

In our sensitivity analysis of identifiability as a model property, we distinguish three different model scenarios. Our first scenario, called BASELINE, corresponds to the original model specification of An & Schorfheide (2007). Accordingly, we consider full inflation indexation and switch off the discount factor shifter. In our second scenario, called PARTIAL INDEXATION, we analyze the effect of adding the partial inflation indexation scheme to the BASELINE scenario. In our third scenario, called PREFERENCE SHOCK, we add the discount factor shifter to the BASELINE model. We run all scenarios under the four different monetary policy rules. We only discuss the results for observable variables YGR_t, $INFL_t$, and INT_t here, as, apart from some steady state parameters like γ_Q that can only be identified from YGR, other combinations of model variables do not change our results significantly. We refer to the replication files for the full set of results, i.e. for all possible combinations of up to three variables. Table 6 summarizes whether the rank requirements are fulfilled for the different scenarios and monetary policy rules. As shown by e.g. Komunjer & Ng (2011) or Qu & Tkachenko (2012), the monetary policy parameters $(\psi_y,\ \psi_\pi,\ \rho_R,\ \sigma_R)$ cannot be identified in the BASELINE specification when using the FLEX-PRICE or the SW monetary rule, whereas in the STEADY STATE or GROWTH specification these parameters are locally identifiable. Our analysis shows two more ways to dissolve the lack of identification, which are, moreover, independent of the functional form of the Taylor rule: adding a preference shock and/or using a partial inflation indexation scheme. Intuitively, this is due to their effect on the transmission channel of monetary policy as noted by e.g. Schmitt-Grohé & Uribe (2004a).

Tables 7 and 8 give insight into the strength of identification according to the Bayesian learning rate indicator of Koop et al. (2013) for the FLEX-PRICE BASELINE, STEADY STATE BASELINE, FLEX-PRICE PREFERENCE SHOCK and FLEX-PRICE INDEXATION scenarios. We

Table 6: Rank checks for monetary model

Scenario	Monetary Policy Specification			
	FLEX-PRICE	STEADY STATE	GROWTH	SW
BASELINE	$[\psi_\pi, \psi_y, \rho_R, \sigma_R]$	✓✓	✓✓	$[\psi_\pi, \psi_y, \rho_R, \sigma_R]$
PARTIAL INDEXATION	✓✓	✓✓	✓✓	✓✓
PREFERENCE SHOCK	✓✓	✓✓	✓✓	✓✓
PARTIAL INDEXATION and PREFERENCE SHOCK	✓✓	✓✓	✓✓	✓✓

Notes: Observable variables are YGR_t, $INFL_t$ and INT_t. All three rank criteria come to the same conclusion, so we do not separately display the result. A ✓✓ indicates that all model parameters are theoretically identifiable, whereas $[\psi_\pi, \psi_y, \rho_R, \rho_\sigma]$ indicates that these parameters cannot be identified jointly.

focus in particular on these scenarios as the non-identified model of An & Schorfheide (2007) corresponds to our FLEX-PRICE BASELINE scenario and the SW rule behaves similarly to the FLEX-PRICE specification. Our simulation and estimation exercise shows that ψ_π and ψ_y are weakly identifiable in the original (theoretically non-identified) model, since the rates at which the posterior precisions are updated are slower than the sample size. In other words, the average posterior precision values in the table tend towards zero instead of constant values. Moreover, estimating non-identified models yields severe problems in the estimation of other, actually identified model parameters. This is accompanied by many difficulties and caveats in the initialization of the proposal distribution for the MCMC algorithm as finding the mode and a positive definite Hessian at the mode is tedious, see A. Albeit, this is an inherent problem of many (even identified) DSGE models, lack of identification of some parameters aggravates this. If, however, the monetary policy authority reacts to deviations from steady state all parameters, including the ones in the Taylor rule, are strongly identified. The same is true for the flex-price Taylor rule, when we introduce a partial inflation indexation scheme. A preference shock, on the other hand, leaves several parameters (ψ_π, ψ_y, ρ_ζ, and σ_ζ) weakly identified.

To sum up, our results provide theoretical support (in terms of identification) for including a partial inflation indexation scheme into modern DSGE models.

Table 7: Average posterior precisions for monetary model

	PARAMETERS															
T	r^A	π^A	γ^Q	τ	ν	ψ_π	ψ_y	ρ_R	ρ_g	ρ_z	$100\sigma_R$	$100\sigma_g$	$100\sigma_z$	ρ_ζ	$100\sigma_\zeta$	ι^p
	BASELINE WITH FLEX-PRICE TAYLOR RULE (*)															
100	0.061	0.270	0.461	0.068	38.082	0.228	2.818	5.493	3.363	29.575	40.687	3.981	11.028	-	-	-
300	0.034	0.146	0.228	0.049	32.200	0.075	1.141	4.722	8.228	19.407	36.607	5.294	10.092	-	-	-
900	0.020	0.085	0.155	0.034	24.358	0.031	0.722	4.746	9.009	19.107	43.514	5.551	11.034	-	-	-
2700	0.018	0.075	0.131	0.036	25.799	0.010	0.262	4.821	8.305	20.746	45.198	5.535	14.533	-	-	-
8100	0.016	0.068	0.121	0.037	27.490	0.004	0.108	4.449	8.183	21.107	43.708	5.567	15.825	-	-	-
	BASELINE WITH STEADY STATE TAYLOR RULE															
100	0.055	0.145	0.444	0.055	40.977	0.279	2.779	4.398	3.865	34.385	40.505	3.206	9.897	-	-	-
300	0.030	0.049	0.226	0.050	31.939	0.101	0.853	3.678	17.043	14.894	35.231	3.558	5.383	-	-	-
900	0.017	0.046	0.145	0.024	18.887	0.075	1.748	4.164	17.572	17.036	43.902	4.465	5.656	-	-	-
2700	0.016	0.045	0.122	0.022	16.903	0.054	1.471	4.197	14.471	17.612	45.664	4.491	5.992	-	-	-
8100	0.015	0.043	0.125	0.021	17.104	0.058	1.778	3.940	13.030	16.057	43.692	4.628	6.020	-	-	-
	PREFERENCE SHOCK WITH FLEX-PRICE TAYLOR RULE (*)															
100	0.063	0.268	0.481	0.087	61.319	0.221	2.025	4.037	2.278	20.611	32.155	1.751	10.793	0.276	0.339	-
300	0.034	0.157	0.241	0.040	32.052	0.077	1.067	4.426	4.122	19.996	35.010	3.069	9.786	0.086	0.241	-
900	0.022	0.092	0.151	0.038	27.964	0.028	0.615	4.716	4.468	20.622	39.890	1.802	10.865	0.031	0.094	-
2700	0.023	0.086	0.153	0.034	22.883	0.010	0.271	4.080	7.164	18.756	41.170	1.228	11.017	0.013	0.055	-
8100	0.017	0.070	0.117	0.036	26.957	0.003	0.084	4.599	8.017	21.936	42.120	1.331	14.445	0.005	0.030	-
	INDEXATION WITH FLEX-PRICE TAYLOR RULE (*)															
100	0.061	0.114	0.451	0.065	47.761	0.280	0.991	7.112	3.136	20.011	38.531	4.058	4.586	-	-	2.998
300	0.035	0.064	0.232	0.050	41.508	0.110	0.443	5.540	7.330	16.998	33.627	5.341	5.440	-	-	2.599
900	0.020	0.035	0.148	0.029	26.825	0.063	0.341	5.555	8.928	18.567	40.908	5.454	4.947	-	-	2.533
2700	0.018	0.030	0.122	0.033	27.352	0.027	0.182	5.442	8.287	21.504	40.337	5.572	6.773	-	-	2.613
8100	0.018	0.032	0.129	0.033	28.766	0.023	0.154	5.070	8.175	22.077	39.142	5.596	6.651	-	-	2.458

Notes: Observable variables are YGR_t, $INFL_t$, and INT_t. Posterior precisions are computed from the draws of the marginal posterior distributions given by a Random-Walk Metropolis-Hastings (RWMH) sampling algorithm based on four Markov chains with each 1000000 draws, half are being discarded as burn-in draws in each chain. The mode and Hessian evaluated at the mode (computed by Dynare's `mode_compute=4`, i.e. Chris Sims's CSMINWEL) are used to determine the initial Gaussian proposal density with scale parameter set such that the acceptance ratios lie in between 20%-35%. A (*) indicates cases where we used an advanced mode finding procedure, as outlined in A.

Table 8: Convergence ratios of posterior precisions for monetary model

	PARAMETERS															
T	r^A	π^A	γ^Q	τ	ν	ψ_π	ψ_y	ρ_R	ρ_g	ρ_z	$100\sigma_R$	$100\sigma_g$	$100\sigma_z$	ρ_ζ	$100\sigma_\zeta$	ι^p
	BASELINE WITH FLEX-PRICE TAYLOR RULE (*)															
300/100	1.664	1.619	1.486	2.190	2.537	0.980	1.214	2.579	7.339	1.969	2.699	3.990	2.746	-	-	-
900/300	1.806	1.747	2.034	2.045	2.269	1.262	1.900	3.015	3.285	2.954	3.566	3.145	3.280	-	-	-
2700/900	2.685	2.650	2.534	3.242	3.178	0.940	1.089	3.048	2.766	3.257	3.116	2.991	3.951	-	-	-
8100/2700	2.618	2.709	2.767	3.050	3.197	1.237	1.233	2.768	2.956	3.052	2.901	3.017	3.267	-	-	-
	BASELINE WITH STEADY STATE TAYLOR RULE															
300/100	1.615	1.020	1.527	2.697	2.338	1.084	0.921	2.509	13.228	1.299	2.609	3.329	1.632	-	-	-
900/300	1.720	2.811	1.929	1.479	1.774	2.248	6.147	3.396	3.093	3.431	3.738	3.765	3.152	-	-	-
2700/900	2.725	2.947	2.524	2.664	2.685	2.137	2.525	3.023	2.470	3.102	3.120	3.018	3.178	-	-	-
8100/2700	2.948	2.878	3.066	2.939	3.036	3.212	3.626	2.817	2.701	2.735	2.870	3.091	3.014	-	-	-
	PREFERENCE SHOCK WITH FLEX-PRICE TAYLOR RULE (*)															
300/100	1.605	1.759	1.504	1.374	1.568	1.048	1.581	3.289	5.427	2.911	3.266	5.257	2.720	0.934	2.129	-
900/300	1.951	1.752	1.883	2.866	2.617	1.083	1.728	3.197	3.252	3.094	3.418	1.762	3.331	1.092	1.176	-
2700/900	3.073	2.805	3.029	2.696	2.455	1.088	1.325	2.595	4.810	2.729	3.096	2.044	3.042	1.223	1.764	-
8100/2700	2.289	2.444	2.307	3.209	3.534	0.924	0.925	3.382	3.357	3.509	3.069	3.251	3.933	1.233	1.616	-
	INDEXATION WITH FLEX-PRICE TAYLOR RULE (*)															
300/100	1.738	1.691	1.541	2.294	2.607	1.177	1.341	2.337	7.011	2.548	2.618	3.949	3.558	-	-	2.601
900/300	1.719	1.642	1.919	1.721	1.939	1.716	2.313	3.008	3.654	3.277	3.650	3.064	2.728	-	-	2.924
2700/900	2.664	2.580	2.466	3.425	3.059	1.307	1.602	2.939	2.785	3.475	2.958	3.065	4.107	-	-	3.095
8100/2700	3.051	3.140	3.177	2.992	3.155	2.509	2.531	2.795	2.960	3.080	2.911	3.013	2.946	-	-	2.822

Notes: Observable variables are YGR_t, $INFL_t$, and INT_t. Posterior precisions are computed from the draws of the marginal posterior distributions given by a Random-Walk Metropolis-Hastings (RWMH) sampling algorithm based on four Markov chains with each 100000 draws, half are being discarded as burn-in draws in each chain. The mode and Hessian evaluated at the mode (computed by Dynare's `mode_compute=4`, i.e. Chris Sims's CSMINWEL) are used to determine the initial Gaussian proposal density with scale parameter set such that the acceptance ratios lie in between 20%-35%. A (*) indicates cases where we used an advanced mode finding procedure, as outlined in A.

5. Discussion of results

First, both theoretical lack of and empirical weak identification are often due to an unfortunate choice of observables. In some cases, like the steady state parameters in our monetary model, this seems obvious. In other cases, like in our investment-adjustment costs model, one should use a specific observable variable (e.g. consumption) instead of other, commonly used ones (e.g. output). In this line of thought, Kim (2003) already pointed out, that information on the relative price of investment can also solve the identification problem, and hence, it is not surprising that current papers include this price in estimated DSGE models, see e.g. Justiniano et al. (2011). As the literature on the choice of observables is still very sparse (Canova et al., 2014; Guerron-Quintana, 2010), we advocate (and show means) to do a brute-force sensitivity analysis before taking a model to actual data. Second, our results show that the identification failure in the Kim (2003) model can be dissolved by specifying intertemporal investment adjustment costs in terms of growth instead of the investment-capital ratio, whereas specifying the output-gap in terms of deviations from steady state or trend-growth identifies the An & Schorfheide (2007) model. Third, by using model features like capital utilization in the first model or partial inflation indexation in the second model, one can identify the models independent of the concrete specification of investment adjustment costs or Taylor rule. Fourth, the same is true when adding an investment-specific technological shock in the former or a preference shock in the latter model.

Our results are relevant from a model building perspective, because it is crucial for macroeconomists to know what frictions and shocks can coexist within models without redundancy. Hence, on the one hand, our finding that the investment-growth specification of intertemporal costs is not subject to functional equivalence with multisectoral costs is useful, especially since this specification is now the benchmark in the quantitative DSGE literature. Accordingly, a recent example is Moura (2018) who is able to use both types of costs to study investment price rigidities in a multisectoral DSGE model. On the other hand, our findings show that by adding different model features and/or shocks one is even able to identify models with both multisectoral and intertemporal level costs. Our results are not limited to investment. Similar specifications are used to model imperfect labor mobility between the consumption-sector and the investment-sector, see e.g. (Nadeau, 2009, Ch.2). Likewise, the monetary policy rule needs to be specified carefully. But our findings show that including a partial inflation indexation scheme provides researchers with more flexibility in the precise shape of the Taylor rule.

6. Conclusion

We strongly recommend that researchers should treat parameter identification as a model property, i.e. from a model building perspective. A wise choice on observables or slight and subtle changes in model assumptions, functional specifications, or structural shocks have an impact on both theoretical (yes/no) identification properties as well as on the strength of identification. In this regard, we side with Adolfson et al. (2019) who argue that "lack of identification should neither be ignored nor be assumed to affect all DSGE models, [...] identification problems can be readily assessed on a case-by-case basis". We extend their approach by using different diagnostic tools for theoretical as well as empirical identification properties and also show means to dissolve the identification failures. Moreover, our paper also has a computational contribution as our research feeds into and extends Dynare's (Adjemian et al., 2011) identification toolbox. In particular, we provide means to analyze the criteria of Komunjer & Ng (2011) and Qu & Tkachenko (2012) by using analytical (instead of numerical) derivatives to compute the relevant Jacobians. Lastly, as our example models are of small scale and easy to replicate and extend, they should be useful for both applied and theoretical macroeconomists as well as for teaching purposes.

References

Adjemian, S., Bastani, H., Juillard, M., Karamé, F., Maih, J., Mihoubi, F., Perendia, G., Pfeifer, J., Ratto, M., & Villemot, S. (2011). *Dynare: Reference Manual Version 4*. Dynare Working Papers 1 CEPREMAP.

Adolfson, M., Laséen, S., Lindé, J., & Ratto, M. (2019). Identification versus misspecification in New Keynesian monetary policy models. *European Economic Review*, *113*, 225–246. doi:10.1016/j.euroecorev.2018.12.010.

An, S., & Schorfheide, F. (2007). Bayesian Analysis of DSGE Models. *Econometric Reviews*, *26*, 113–172. doi:10.1080/07474930701220071.

Andreasen, M. M. (2010). How to Maximize the Likelihood Function for a DSGE Model. *Computational Economics*, *35*, 127–154. doi:10.1007/s10614-009-9182-6.

Canova, F., Ferroni, F., & Matthes, C. (2014). Choosing the variables to estimate singular DSGE models. *Journal of Applied Econometrics*, *29*, 1099–1117. doi:10.1002/jae.2414.

Canova, F., & Sala, L. (2009). Back to square one: Identification issues in DSGE models. *Journal of Monetary Economics*, *56*, 431–449. doi:10.1016/j.jmoneco.2009.03.014.

Chadha, J., & Shibayama, K. (2018). *Bayesian Estimation of DSGE models: Identification using a diagnostic indicator*. Working Paper NIESR and University of Kent.

Christiano, L. J., Eichenbaum, M., & Evans, C. L. (2005). Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy*, *113*, 1–45. doi:10.1086/426038.

Fernández-Villaverde, J., Rubio-Ramírez, J. F., & Schorfheide, F. (2016). Solution and Estimation Methods for DSGE Models. In J. B. Taylor, & H. Uhlig (Eds.), *Handbook of Macroeconomics* (pp. 527–724). North-Holland volume 2. doi:10.1016/bs.hesmac.2016.03.006.

Greenwood, J., Hercowitz, Z., & Krusell, P. (2000). The role of investment-specific technological change in the business cycle. *European Economic Review*, *44*, 91–115. doi:10.1016/S0014-2921(98)00058-0.

Guerron-Quintana, P. A. (2010). What you match does matter: the effects of data on DSGE estimation. *Journal of Applied Econometrics*, *25*, 774–804. doi:10.1002/jae.1106.

Huffman, G. W., & Wynne, M. A. (1999). The role of intratemporal adjustment costs in a multisector economy. *Journal of Monetary Economics*, *43*, 317–350. doi:10.1016/S0304-3932(98)00059-2.

Iskrev, N. (2010). Local identification in DSGE models. *Journal of Monetary Economics*, *57*, 189–202. doi:10.1016/j.jmoneco.2009.12.007.

Justiniano, A., Primiceri, G. E., & Tambalotti, A. (2010). Investment shocks and business cycles. *Journal of Monetary Economics*, *57*, 132–145. doi:10.1016/j.jmoneco.2009.12.008.

Justiniano, A., Primiceri, G. E., & Tambalotti, A. (2011). Investment shocks and the relative price of investment. *Review of Economic Dynamics*, *14*, 102–121. doi:10.1016/j.red.2010.08.004.

Kamber, G., McDonald, C., Sander, N., & Theodoridis, K. (2016). Modelling the business cycle of a small open economy: The Reserve Bank of New Zealand's DSGE model. *Economic Modelling*, *59*, 546–569. doi:10.1016/j.econmod.2016.08.013.

Kim, J. (2003). Functional Equivalence between Intertemporal and Multisectoral Investment Adjustment Costs. *Journal of Economic Dynamics and Control*, *27*, 533–549. doi:10.1016/s0165-1889(01)00060-4.

Komunjer, I., & Ng, S. (2011). Dynamic Identification of Dynamic Stochastic General Equilibrium Models. *Econometrica*, *79*, 1995–2032. doi:10.3982/ECTA8916.

Koop, G., Pesaran, M. H., & Smith, R. P. (2013). On Identification of Bayesian DSGE Models. *Journal of Business & Economic Statistics*, *31*, 300–314. doi:10.1080/07350015.2013.773905.

Lucas, R. E., & Prescott, E. C. (1971). Investment under Uncertainty. *Econometrica*, *39*, 659–681. doi:10.2307/1909571.

Morris, S. D. (2017). DSGE pileups. *Journal of Economic Dynamics and Control*, *74*, 56–86. doi:10.1016/j.jedc.2016.11.002.

Moura, A. (2018). Investment shocks, sticky prices, and the endogenous relative price of investment. *Review of Economic Dynamics*, *27*, 48–63. doi:10.1016/j.red.2017.11.004.

Mutschler, W. (2015). Identification of DSGE models – The effect of higher-order approximation and pruning. *Journal of Economic Dynamics and Control*, *56*, 34–54. doi:10.1016/j.jedc.2015.04.007.

Mutschler, W. (2016). *Local identification of nonlinear and non-Gaussian DSGE models*. Number 10 in Wissenschaftliche Schriften der WWU Münster, Reihe IV. Münster: Monsenstein und Vannerdat. PhD Thesis.

Mutschler, W. (2019). Improvement of Identification Toolbox (Merge Request). https://git.dynare.org/Dynare/dynare/merge_requests/1648. [Online; accessed May-7-2019].

Nadeau, J.-F. (2009). *Essays in Macroeconomics*. PhD Thesis University of Britisch Columbia Vancouver.

Qu, Z., & Tkachenko, D. (2012). Identification and frequency domain quasi-maximum likelihood estimation of linearized dynamic stochastic general equilibrium models. *Quantitative Economics*, *3*, 95–132. doi:10.3982/QE126.

Ratto, M., & Iskrev, N. I. (2011). *Identification analysis of DSGE models with DYNARE*. MONFISPOL 225149 European Commission and Banco de Portugal.

Ríos-Rull, J.-V., Schorfheide, F., Fuentes-Albero, C., Kryshko, M., & Santaeulália-Llopis, R. (2012). Methods versus substance: Measuring the effects of technology shocks. *Journal of Monetary Economics*, *59*, 826–846. doi:10.1016/j.jmoneco.2012.10.008.

Schmitt-Grohé, S., & Uribe, M. (2004a). Optimal fiscal and monetary policy under sticky prices. *Journal of Economic Theory*, *114*, 198–230. doi:10.1016/S0022-0531(03)00111-X.

Schmitt-Grohé, S., & Uribe, M. (2004b). *Optimal Operational Monetary Policy in the Christiano-Eichenbaum-Evans Model of the U.S. Business Cycle*. Working Paper 10724 National Bureau of Economic Research. doi:10.3386/w10724.

Smets, F., & Wouters, R. (2007). Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *American Economic Review*, *97*, 586–606. doi:10.1257/aer.97.3.586.

A. Posterior mode finding

We use an advanced mode finding procedure, where we sequentially loop over different optimization algorithms taking the previous found mode as initial value for the next optimizer. In particular, we loop, in this order, over Dynare's `mode_compute` values equal to 9 (CMA-ES), 8 (Nelson-Mead Simplex), 4 (csminwel), 7 (fminsearch) and 1 (fmincon). We then rely on Dynare's (very time-consuming) `mode_compute=6` optimizer, i.e. a "Monte Carlo Optimizer" to get a well-behaved Hessian in the relevant parameter space. The intuition is that the Metropolis-Hastings algorithm does not need to start from the posterior mode to converge to the posterior distribution. It is only required to start from a point with a high posterior density value and to use an estimate of the covariance matrix for the jumping distribution (actually any positive definite matrix will suffice). As a side note, the replication files of An & Schorfheide (2007) reveal that they face the same problem in their estimation and overcome this by using different step sizes for the numerical evaluation of second derivatives of the log-likelihood function.